Online Appendix for
“Export policy for dual-use goods”

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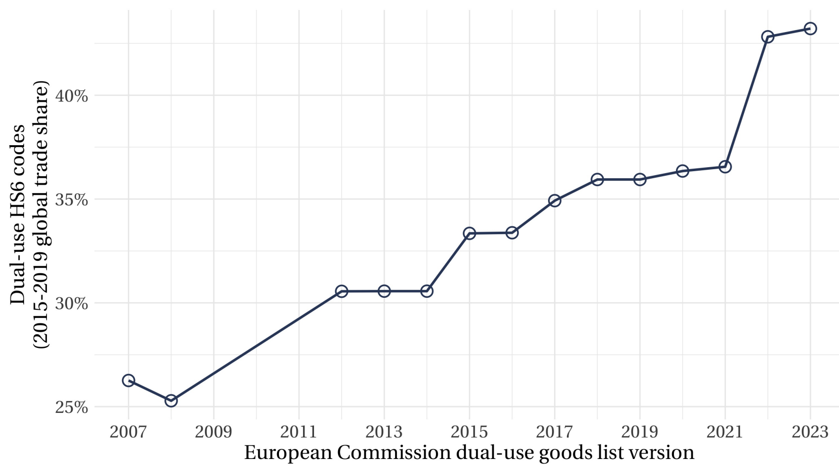
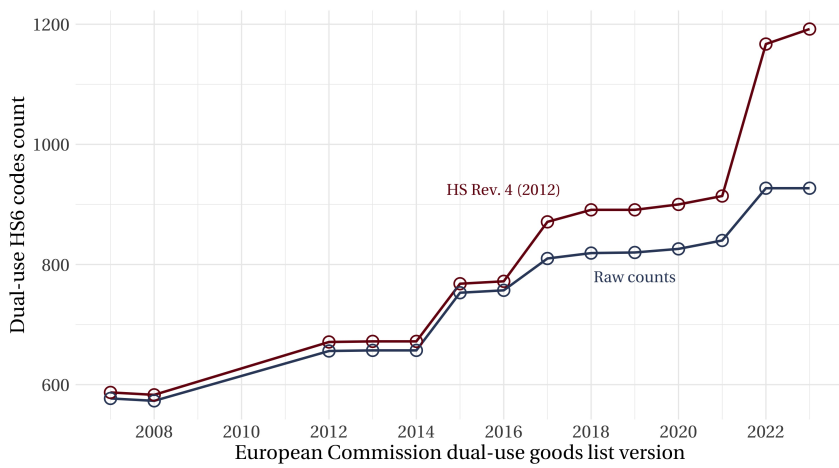
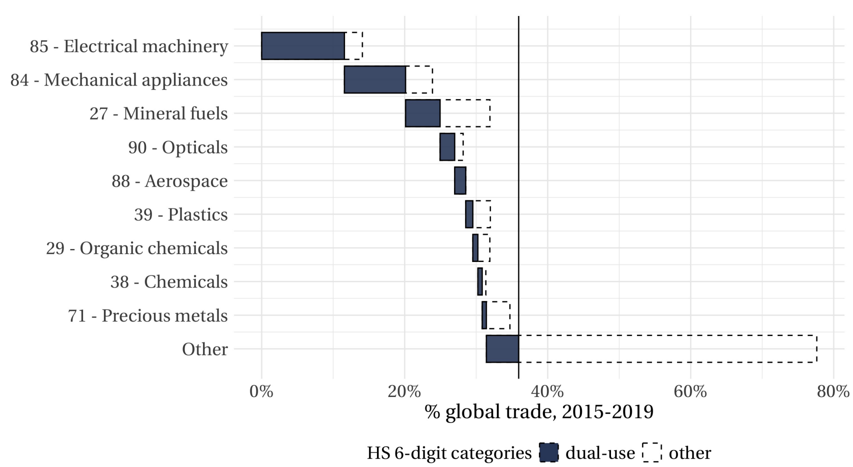
A Introduction



(a) Global trade share share



(b) 2012 Revision



(c) EU dual-use list, 2018: Breakdown by HS2 categories

B Theoretical framework

B.1 Optimal trade policy

Proposition B.1. *The trade taxes for country $i \in \{H, F\}$ in the Nash equilibrium satisfy*

$$\tau_{ik}^{\mathcal{M}} = 1, \quad k \in \{H, F\}, \tag{1}$$

$$\frac{\tau_{-i,i}^{\mathcal{X}} - 1}{\tau_{-i,i}^{\mathcal{X}}} = -\frac{1 + \left(\frac{M_i}{M_{-i}}\right) S_{-i,i}^{M}}{\mathcal{E}_{-i,i}^{-i,i} - 1}, \quad S_{ik}^{M} \equiv \frac{s_{ik}^{M} M_i}{s_{ik}^{C} C_i + s_{ik}^{M} M_i}, \tag{2}$$

where elasticity $\mathcal{E}_{-i,i}^{-i,i} \equiv d \log E_{-i,i} / d \log \tau_{-i,i}^{\mathcal{X}}$ is the import demand elasticity.

The proof of the proposition is given in the Appendix of the main text. These formulas have multiple interpretations that nest the cases of other trade instruments. First, export taxes can be viewed as export-control checks in which goods continue their route but the customs costs are transferred to households. The theory also nests the case of deterministic screening where military and civilian varieties receive separate taxes. Second, export taxes can be implemented through licensing that fixes the quantity of the exported good; quotas can be reinterpreted as a revenue loss that drives the $\mathcal{T}$ terms to zero or to a fraction of their tax values. Export wedges can also be interpreted as randomly disallowing and destroying exports of $1/\tau$ varieties from a continuum crossing the border. Third, the problem of a company exporting dual-use goods abroad is equivalent to the problem of the government setting export policy, to the extent that the government compels the company to internalize the externality.

The final case to consider is that of discrete bans, which impose infinite rather than finite taxes. As a reminder, the optimal welfare can be expressed as

$$W_i = w_i L_i + \frac{(P_i^C)^{1-\eta_i}}{\eta_i - 1} + \frac{(P_{-i}^C)^{1-\eta_{-i}}}{\eta_{-i} - 1} + \beta_i^{\zeta_i} M_{-i}^{1-\zeta_i} \frac{(P_i^M / P_{-i}^M)^{1-\zeta_i}}{\zeta_i - 1}. \tag{3}$$

Assuming for simplicity a global economy with a CES nested structure with elasticity σ, the welfare changes are

$$U_j^{C\prime} = U_j^C [\mathbf{s_{j\cdot}^C} \mathbf{\Psi_{\cdot k}} \Omega_{k,-i}]^{\frac{1}{1-\sigma}}, \quad \Delta U_i^{M\prime} = U_i^M \left[\frac{\mathbf{s_{i\cdot}^{M\prime}} \mathbf{\Psi_{\cdot k}} \Omega_{k,-i}}{\mathbf{s_{-i\cdot}^{M}}{}' \mathbf{\Psi_{\cdot k}} \Omega_{k,-i}} \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

so that the key sufficient statistic for trade-offs is now

$$\frac{1}{\sigma - 1} \ln \mathcal{C}_{ik}^{M} \equiv \frac{\mathcal{C}_{ik}^{M}}{\sigma} \left[\frac{\sigma}{\sigma - 1} \frac{\ln \mathcal{C}_{ik}^{M}}{\mathcal{C}_{ik}^{M}} \right], \tag{5}$$

which contains largely the same terms as before. Baqaee et al. (2023) provide more sophisticated accounting for the disappearance of varieties in the network context. In contrast, we stress-test that sufficient statistic in our empirical section and find that our baseline measure delivers better performance.

We now proceed to the case of a sequential game in which trade policy is chosen before defense spending. Such a game presents a reduced-form way of modeling dynamics if one believes that trade policy choices made today can affect military build-up tomorrow (e.g. through resource stockpiling, military investment, or delays in observing foreign military strategies).

Proposition B.2 (Sequential game)**.** *Consider the game in which governments set trade policies* $(\mathcal{P}_\tau^{(H)}, \mathcal{P}_\tau^{(F)})$ *first and defense spending* $(\mathcal{P}_M^{(H)}, \mathcal{P}_M^{(F)})$ *second. The trade taxes for country* $i \in \{H, F\}$ *in the subgame perfect Nash equilibrium are characterized by*

$$\frac{\tau^{\mathcal{X}}_{-i,i} - 1}{\tau^{\mathcal{X}}_{-i,i}} = -\frac{\mathcal{T}^{\mathcal{X}}_{-i,i} + \zeta_{i,-i}\tau^{\mathcal{M}}_{-i,i}(M_i/M_{-i})S^M_{-i,i}}{\mathcal{E}^{-i,i}_{-i,i} - 1}, \tag{5}$$

$$\frac{\tau^{\mathcal{M}}_{ik} - 1}{\tau^{\mathcal{M}}_{ik}} = -\frac{\mathcal{T}^{\mathcal{M}}_{ik} + (1 - \zeta_{i,-i})S^M_{ik}}{\mathcal{E}^{ik}_{ik} - 1}, \quad k \in \{H, F\}, \tag{5}$$

where $\mathcal{T}^{\mathcal{X}}$ *and* $\mathcal{T}^{\mathcal{M}}$ *are the usual terms-of-trade components featuring revenue spillovers following trade diversion, and* $\zeta_{i,-i} \equiv \zeta_{-i}/(\zeta_i + \zeta_{-i} - \zeta_i\zeta_{-i})$ *is the conflict elasticity. The terms-of-trade components can be expanded as*

$$\mathcal{T}^{\mathcal{X}}_{-i,i} \equiv 1 + (E_{-i,i}/\tau^{\mathcal{M}}_{-i,i})^{-1} \sum_{k \in \{H,F\}} \frac{\tau^{\mathcal{M}}_{ik} - 1}{\tau^{\mathcal{M}}_{ik}} E_{ik}\mathcal{E}^{ik}_{-i,i}, \tag{5}$$

$$\mathcal{T}^{\mathcal{M}}_{ik} \equiv E_{ik}^{-1}\left[\frac{\tau^{\mathcal{X}}_{-i,i} - 1}{\tau^{\mathcal{X}}_{-i,i}\tau^{\mathcal{M}}_{-i,i}} E_{-i,i}\mathcal{E}^{-i,i}_{ik} + \frac{\tau^{\mathcal{M}}_{ik} - 1}{\tau^{\mathcal{M}}_{ik}} E_{ik}\mathcal{E}^{-i,i}_{ik}\right]. \tag{5}$$

Proof. The welfare function is given by

$$W_i = wL_i + R_i + \frac{C_i}{\eta_i - 1} + \frac{M_i}{\zeta_i - 1}. \tag{6}$$

We express the welfare change following small changes in trade taxes, using

$$d\log C_i = (1 - \eta_i)d\log P^C_i, \tag{7}$$

$$d\log M_i = (1 - \zeta_i)\zeta_{i,-i}(d\log P^M_i - d\log P^M_{-i}), \tag{8}$$

we rewrite the welfare change as

$$dW_i = dR_i + M_i\zeta_{i,-i}d\log P^M_{-i} - C_i d\log P^C_i - M_i\zeta_{i,-i}d\log P^M_i. \tag{9}$$

The price changes are the same; however, the revenue changes are different.

The revenue changes are

$$dR_i = \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}\tau^{\mathcal{X}}_{-i,i}} d\log\tau^{\mathcal{X}}_{-i,i} + \frac{(\tau^{\mathcal{X}}_{-i,i}-1)E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}\tau^{\mathcal{X}}_{-i,i}} d\log E_{-i,i} \tag{10}$$

$$+ \sum_{k\in\{H,F\}} \frac{E_{ik}}{\tau^{\mathcal{M}}_{ik}} d\log\tau^{\mathcal{M}}_{ik} + \frac{(\tau^{\mathcal{M}}_{ik}-1)E_{ik}}{\tau^{\mathcal{M}}_{ik}} d\log E_{ik}. \tag{11}$$

We express changes in procurement as

$$d\log E_{-i,i} = \mathcal{E}^{-i,i}_{-i,i} d\log\tau^{\mathcal{X}}_{-i,i} + \sum_{k\in\mathcal{K}} \mathcal{E}^{-i,i}_{ik} d\log\tau^{\mathcal{M}}_{ik}, \tag{12}$$

$$d\log E_{ik} = \mathcal{E}^{ik}_{-i,i} d\log\tau^{\mathcal{X}}_{-i,i} + \sum_{l\in\mathcal{K}} \mathcal{E}^{ik}_{il} d\log\tau^{\mathcal{M}}_{il}, \tag{13}$$

where

$$\mathcal{E}^{-i,i}_{-i,i} = S^C_{-i,i}s^C_{-i,i}(1-\xi_{-i}) + S^M_{-i,i}s^M_{-i,i}(1-\zeta_{-i})\zeta_{-i,i}, \tag{14}$$

$$\mathcal{E}^{-i,i}_{ik} = -S^M_{-i,i}s^M_{ik}(1-\zeta_{-i})\zeta_{-i,i}, \tag{15}$$

$$\mathcal{E}^{ik}_{-i,i} = -S^M_{ik}s^M_{-i,i}(1-\zeta_i)\zeta_{i,-i}, \tag{16}$$

$$\mathcal{E}^{ik}_{il} = S^C_{ik}s^C_{il}(1-\xi_i) + S^M_{ik}s^M_{il}(1-\zeta_i)\zeta_{i,-i}. \tag{17}$$

The changes in revenues can be thus recast as

$$dR_i = \left\{ \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} \frac{\tau^{\mathcal{X}}_{-i,i}-1}{\tau^{\mathcal{X}}_{-i,i}} (\mathcal{E}^{-i,i}_{-i,i}-1) + \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} + \sum_{k\in\{H,F\}} \frac{\tau^{\mathcal{M}}_{ik}-1}{\tau^{\mathcal{M}}_{ik}} E_{ik}\mathcal{E}^{ik}_{-i,i} \right\} d\log\tau^{\mathcal{X}}_{-i,i} \tag{18}$$

$$+ \sum_{k\in\{H,F\}} \left\{ E_{ik} + \frac{\tau^{\mathcal{M}}_{ik}-1}{\tau^{\mathcal{M}}_{ik}} E_{ik}(\mathcal{E}^{ik}_{ik}-1) + \sum_{l\neq k} \frac{\tau^{\mathcal{M}}_{il}-1}{\tau^{\mathcal{M}}_{il}} E_{il}\mathcal{E}^{il}_{ik} + \frac{\tau^{\mathcal{X}}_{-i,i}-1}{\tau^{\mathcal{X}}_{-i,i}} \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} \mathcal{E}^{-i,i}_{ik} \right\} d\log\tau^{\mathcal{M}}_{ik}$$

Collecting revenue changes and price changes into welfare changes yields

$$dW_i = \left\{ \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} \frac{\tau^{\mathcal{X}}_{-i,i}-1}{\tau^{\mathcal{X}}_{-i,i}} (\mathcal{E}^{-i,i}_{-i,i}-1) + \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} + \sum_{k\in\{H,F\}} \frac{\tau^{\mathcal{M}}_{ik}-1}{\tau^{\mathcal{M}}_{ik}} E_{ik}\mathcal{E}^{ik}_{-i,i} + M_i s^M_{-i,i}\zeta_{i,-i} \right\} d\log\tau^{\mathcal{X}}_{-i,i} \tag{19}$$

$$+ \sum_{k\in\{H,F\}} \left\{ \frac{\tau^{\mathcal{M}}_{ik}-1}{\tau^{\mathcal{M}}_{ik}} E_{ik}(\mathcal{E}^{ik}_{ik}-1) + \sum_{l\neq k} \frac{\tau^{\mathcal{M}}_{il}-1}{\tau^{\mathcal{M}}_{il}} E_{il}\mathcal{E}^{il}_{ik} + \frac{\tau^{\mathcal{X}}_{-i,i}-1}{\tau^{\mathcal{X}}_{-i,i}} \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} \mathcal{E}^{-i,i}_{ik} + M_i s^M_{ik}(1-\zeta_{i,-i}) \right\} d\log\tau^{\mathcal{M}}_{ik}$$

The best response is characterized by the matrix equation

$$\begin{bmatrix} \frac{\tau^{\mathcal{X}}_{-i,i}-1}{\tau^{\mathcal{X}}_{-i,i}} \\ \frac{\tau^{\mathcal{M}}_{ii}-1}{\tau^{\mathcal{M}}_{ii}} \\ \frac{\tau^{\mathcal{M}}_{i,-i}-1}{\tau^{\mathcal{M}}_{i,-i}} \end{bmatrix} = - \begin{bmatrix} \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}}(\mathcal{E}^{-i,i}_{-i,i}-1) & E_{ii}\mathcal{E}^{ii}_{-i,i} & E_{i,-i}\mathcal{E}^{i,-i}_{-i,i} \\ \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}}\mathcal{E}^{-i,i}_{ii} & E_{ii}(\mathcal{E}^{ii}_{ii}-1) & E_{i,-i}\mathcal{E}^{i,-i}_{ii} \\ \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}}\mathcal{E}^{-i,i}_{i,-i} & E_{ii}\mathcal{E}^{ii}_{i,-i} & E_{i,-i}(\mathcal{E}^{i,-i}_{i,-i}-1) \end{bmatrix}^{-1} \begin{bmatrix} \frac{E_{-i,i}}{\tau^{\mathcal{M}}_{-i,i}} \left(1 + \tau^{\mathcal{M}}_{-i,i}\zeta_{i,-i}\frac{M_i}{M_{-i}}S^M_{-i,i}\right) \\ E_{ii}(1-\zeta_{i,-i})S^M_{ii} \\ E_{i,-i}(1-\zeta_{i,-i})S^M_{i,-i} \end{bmatrix} \tag{20}$$

Another way to write the optimal taxes is

$$\frac{\tau^{\mathcal{X}}_{-i,i}-1}{\tau^{\mathcal{X}}_{-i,i}} = -\frac{\mathcal{T}^{\mathcal{X}}_{-i,i}+\tau^{\mathcal{M}}_{-i,i}\zeta_{i,-i}(M_i/M_{-i})S^M_{-i,i}}{\mathcal{E}^{-i,i}_{-i,i}-1}, \tag{21}$$

$$\frac{\tau^{\mathcal{M}}_{ik}-1}{\tau^{\mathcal{M}}_{ik}} = -\frac{\mathcal{T}^{\mathcal{M}}_{ik}+(1-\zeta_{i,-i})S^M_{ik}}{\mathcal{E}^{ik}_{ik}-1}, \tag{22}$$

where

$$\mathcal{T}^{\mathcal{X}}_{-i,i} = 1+(E_{-i,i}/\tau^{\mathcal{M}}_{-i,i})^{-1}\sum_{k\in\{H,F\}} R^{\mathcal{M}}_{ik}\mathcal{E}^{ik}_{-i,i}, \tag{23}$$

$$\mathcal{T}^{\mathcal{M}}_{ik} = E^{-1}_{ik}R^{\mathcal{X}}_{-i,i}\mathcal{E}^{-i,i}_{ik}+E^{-1}_{ik}R^{\mathcal{M}}_{ik}\mathcal{E}^{-i,i}_{ik}. \tag{24}$$

□

In that toy extension, in addition to correcting the Marshallian externality, trade interventions now have an additional strategic dimension. Export taxes and domestic subsidies act as deterrents, tilting the price ratio to affect the second stage of the game. The welfare outcome in the first stage is now characterized by

$$dW_i = dR_i + M_i\zeta_{i,-i}d\log P^M_{-i} - C_i d\log P^C_i - M_i\zeta_{i,-i}d\log P^M_i. \tag{25}$$

The degree of the strategic force is characterized by the conflict elasticity $\zeta_{i,-i}$, which reflects how sensitive foreign military spending is to the military price ratio. Under $\zeta_{i,-i}=1$, when the nominal military spending is not sensitive to the price ratio, Propositions B.1 and B.2 yield the same formulas.

Under $\zeta_{i,-i}>1$, when nominal military spending decreases as the price ratio moves unfavorably, strategic incentives both amplify export taxes and generate domestic subsidies.[1] Another way to interpret this is to recognize that the strategic force modifies the macro shifter (M_i/M_{-i}) from Proposition B.1, while keeping the sectoral shifters S^M intact.[2] Proposition B.2 thus demonstrates how dynamic incentives can make a case for trade policy as a strategic deterrent.

B.2 Military centrality in production networks

Before characterizing the optimal trade policy, we introduce some network definitions. We begin with standard definitions of the *Leontief* and *inverse Leontief matrices* and present

[1] Under $\zeta_{i,-i}<1$, the foreign government decreases its military spending when the military price ratio becomes more favorable to it. For the home government, taxing home military goods becomes the optimal policy, as it both raises domestic revenues and deters foreign military spending.

[2] It also modifies terms-of-trade components. This occurs because trade policy now affects final demand, which means that taxes make import demand elasticities, which previously were zero, non-zero. (For example, import taxes in a country now affect export flows.)

some helpful facts about them. Then, we introduce the concepts of *pull weights* and a *distortion matrix*. We use these concepts to introduce *military centrality*, which is the main focus of our analysis.

Definition 1 (Leontief matrices)**.** *The cost-based Leontief matrix* is $\mathbf{\Omega} = (\Omega_{kl})$. *The revenue-based Leontief matrix* is $\tilde{\mathbf{\Omega}} = (\tilde{\Omega}_{kl})$, $\tilde{\Omega}_{kl} = \Omega_{kl}/(\tau^{\mathcal{X}}_{kl}\tau^{\mathcal{M}}_{kl})$.

Definition 2 (Inverse Leontief matrices)**.** *The inverse cost-based Leontief matrix* $\mathbf{\Psi} = (\Psi_{kl})$ and *the inverse revenue-based Leontief matrix* $\tilde{\mathbf{\Psi}} = (\tilde{\Psi}_{kl})$ are defined as

$$\mathbf{\Psi} \equiv (\mathbf{I} - \mathbf{\Omega})^{-1}, \quad \tilde{\mathbf{\Psi}} \equiv (\mathbf{I} - \tilde{\mathbf{\Omega}})^{-1}. \tag{26}$$

The following two facts about Leontief matrices will be helpful for subsequent definitions. First, all the elements of the inverse Leontief matrices Ψ_{kl}, $\tilde{\Psi}_{kl}$ are non-negative, since $\mathbf{\Psi} = \sum_{n=0}^{\infty} \mathbf{\Omega}^n$, $\tilde{\mathbf{\Psi}} = \sum_{n=0}^{\infty} \tilde{\mathbf{\Omega}}^n$. Second, one can rewrite the market clearing condition for goods as

$$\mathbf{X} = \tilde{\mathbf{\Psi}}' \sum_{i \in \{H,F\}} \tilde{\mathbf{E}}_i. \tag{27}$$

One can see it by multiplying both sides of equation (**??**) by p_k and recasting those in the matrix form:

$$X_k = \sum_{l=1}^{K} \tilde{\Omega}_{lk} X_l + \sum_{i \in \{H,F\}} \tilde{E}_{ik}, \quad \mathbf{X} = \tilde{\mathbf{\Omega}}' \mathbf{X} + \sum_{i \in \{H,F\}} \tilde{\mathbf{E}}_i. \tag{28}$$

These facts will be helpful for the next two definitions.

Definition 3 (Final demand weights)**.** *Final demand weights* for firm k from expenditures of country j on firm l's output are

$$\omega^{(j)}_{kl} \equiv \frac{\tilde{E}_{jl}\tilde{\Psi}_{lk}}{X_k} \tag{29}$$

Intuitively, $\omega^{(j)}_{kl}$ is a network-adjusted sales share that goes to country j through final demand for firm l's goods; $\sum_{l \in \mathcal{K}} \omega^{(j)}_{kl}$ is the overall sales share to country j, and $\sum_{j \in \{H,F\}} \sum_{l \in \mathcal{K}} \omega^{(j)}_{kl} = 1$ represents the total sales share, which must sum to 1. This can be verified by observing that

$$X_k = \sum_{j \in \{H,F\}} \sum_{l=1}^{K} \tilde{E}_{jl}\tilde{\Psi}_{lk} \quad \Rightarrow \quad \sum_{j \in \{H,F\}} \sum_{l \in \mathcal{K}} \omega^{(j)}_{kl} = 1. \tag{30}$$

Definition 4 (Distortion matrix)**.** *Distortion matrix* $\boldsymbol{\delta}^{(j)} = (\delta^{(j)}_{kl})$ is defined as

$$\delta^{(j)}_{kl} \equiv \frac{\tau_{jl}\Psi_{lk}}{\tilde{\Psi}_{lk}}, \quad \tau_{jl} \equiv \tau^{\mathcal{X}}_{jl}\tau^{\mathcal{M}}_{jl}. \tag{31}$$

The distortion matrix equals the matrix of ones when there are no taxes. In the economy with non-negative taxes, distortions are all greater than or equal to 1. In the economy with non-negative subsidies, distortions are all less than or equal to 1. These two statements can be verified by showing that

$$\boldsymbol{\Psi} - \tilde{\boldsymbol{\Psi}} = \boldsymbol{\Psi}(\boldsymbol{\Omega} - \tilde{\boldsymbol{\Omega}})\tilde{\boldsymbol{\Psi}}. \tag{32}$$

After introducing these definitions, we proceed with our concepts of firm-level *centrality.*

Definition 5 (Centrality)**.** We define distortion centrality, consumption centrality, and military centrality of firm k for country j as

$$\mathcal{C}^D_{jk} \equiv \sum_{l\in\mathcal{K}_j} \omega^{(j)}_{kl}\delta^{(j)}_{kl}, \tag{33}$$

$$\mathcal{C}^C_{jk} = \sum_{l\in\mathcal{K}_j} \omega^{(j)}_{kl}\delta^{(j)}_{kl}S^C_{jl}, \qquad S^C_{ik} \equiv \frac{s^C_{ik}C_i}{s^C_{ik}C_i + s^M_{ik}M_i}, \tag{34}$$

$$\mathcal{C}^M_{jk} = \sum_{l\in\mathcal{K}_j} \omega^{(j)}_{kl}\delta^{(j)}_{kl}S^M_{jl}, \qquad S^M_{ik} \equiv \frac{s^M_{ik}M_i}{s^C_{ik}C_i + s^M_{ik}M_i}. \tag{35}$$

Intuitively, $\omega^{(j)}_{kl}$ stands for the network adjustment, and $\delta^{(j)}_{kl}$ for the taxation adjustment. An alternative interpretation of these definitions is that nodes with some final sales to country j have a military sales share characteristic S^M_{jl} such that $S^M_{jl} + S^C_{jl} = 1$. The pull weights $\boldsymbol{\omega}$ and the distortion matrix $\boldsymbol{\delta}$ amplify these characteristics:

$$\boldsymbol{\mathcal{C}}^D_j \equiv (\boldsymbol{\omega}^{(j)} \otimes \boldsymbol{\delta}^{(j)})\mathbf{1}, \quad \boldsymbol{\mathcal{C}}^M_j \equiv (\boldsymbol{\omega}^{(j)} \otimes \boldsymbol{\delta}^{(j)})\mathbf{S}^M_j, \quad \boldsymbol{\mathcal{C}}^C_j \equiv (\boldsymbol{\omega}^{(j)} \otimes \boldsymbol{\delta}^{(j)})\mathbf{S}^C_j. \tag{36}$$

One can see that the sum of consumption and military centralities yield distortion centrality:

$$\mathcal{C}^C_{jk} + \mathcal{C}^M_{jk} = \mathcal{C}^D_{jk}. \tag{37}$$

In an economy with no taxes, distortion centrality equals a network-adjusted sales share to a given country, $\sum_{l\in\mathcal{K}} \omega^{(j)}_{kl} \leq 1$. In a closed economy with no taxes, distortion centrality equals 1. The following lemma provides a more intuitive way to express these centrality measures.

Lemma 1 (Centrality equivalence)**.** *Centrality can be restated as*

$$\mathcal{C}^M_{jk} = \frac{[\boldsymbol{\Psi}'\mathbf{s}^{\mathbf{M}}]_{jk}M_j}{[\tilde{\boldsymbol{\Psi}}'\mathbf{s}^{\mathbf{M}}]_{jk}M_j + [\tilde{\boldsymbol{\Psi}}'\mathbf{s}^{\mathbf{C}}]_{jk}C_j}, \qquad \mathcal{C}^C_{jk} = \frac{[\boldsymbol{\Psi}'\mathbf{s}^{\mathbf{C}}]_{jk}C_j}{[\tilde{\boldsymbol{\Psi}}'\mathbf{s}^{\mathbf{M}}]_{jk}M_j + [\tilde{\boldsymbol{\Psi}}'\mathbf{s}^{\mathbf{C}}]_{jk}C_j}. \tag{38}$$

Proof.

$$\mathcal{C}^M_{jk} = \sum_l \frac{E_{jl}\Psi_{lk}}{X_k}S^M_{jl} = \frac{[\boldsymbol{\Psi}'\mathbf{s}^{\mathbf{M}}]_{jk}M_j}{[\tilde{\boldsymbol{\Psi}}'\mathbf{s}^{\mathbf{M}}]_{jk}M_j + [\tilde{\boldsymbol{\Psi}}'\mathbf{s}^{\mathbf{C}}]_{jk}C_j}. \tag{39}$$

As such, in an economy with no taxes, $\boldsymbol{\Psi} = \tilde{\boldsymbol{\Psi}} \Rightarrow \mathcal{C}^M_{jk} \in [0,1]$. □

Another property of this centrality measure is rank invariance in a constant-returns-to-scale economy conditional on factor prices and trade taxes. Regardless of how one scales final agents' incomes, the relative rankings of firms remain the same. This property is helpful for empirical analysis.

Lemma 2 (Rank invariance)**.** *Consider two economies $\mathcal{A}'$, $\mathcal{A}''$ with identical factor prices and no taxation but different values of final demand M and C (e.g., driven by external endowments). Then, for any two industries k and l,*

$$\mathcal{C}_{jk}^{M'} \geq \mathcal{C}_{jl}^{M'} \qquad \Leftrightarrow \qquad \mathcal{C}_{jk}^{M''} \geq \mathcal{C}_{jl}^{M''}$$

Proof. The rankings of centrality are the same as the rankings of military specialization:

$$\mathcal{C}_k^M \geq \mathcal{C}_l^M \quad \Leftrightarrow \quad \frac{1}{1+([\mathbf{\Psi}'\mathbf{s}^{\mathbf{C}}]_k/[\mathbf{\Psi}'\mathbf{s}^{\mathbf{M}}]_k)(C/M)} \geq \frac{1}{1+([\mathbf{\Psi}'\mathbf{s}^{\mathbf{C}}]_l/[\mathbf{\Psi}'\mathbf{s}^{\mathbf{M}}]_l)(C/M)}. \tag{40}$$

The latter inequality can be recast as

$$\frac{[\mathbf{\Psi}'\mathbf{s}^{\mathbf{C}}]_k}{[\mathbf{\Psi}'\mathbf{s}^{\mathbf{M}}]_k} \leq \frac{[\mathbf{\Psi}'\mathbf{s}^{\mathbf{C}}]_l}{[\mathbf{\Psi}'\mathbf{s}^{\mathbf{M}}]_l}. \tag{41}$$

The terms here depend only on the network structure but not on the final demand M and C. Hence, centrality rankings are invariant to the scale of final demand as long as factor prices are kept constant. □

After having defined and explored our centrality concepts, we can proceed with the proposition for the optimal network taxes. (Details of the proof are relegated to Supplementary Appendix A.3.)

Proposition B.3. *The trade taxes for country $i \in \{H, F\}$ and firm $k \in \mathcal{K}_i$ in the Nash equilibrium satisfy*

$$\frac{\tau_{-i,k}^{\mathcal{X}}-1}{\tau_{-i,k}^{\mathcal{X}}} = -\frac{\overbrace{\mathcal{T}_{-i,k}^{\mathcal{X}}}^{\text{ToT}} + \overbrace{\tau_{-i,k}^{\mathcal{M}}\left[\left(\frac{M_i}{M_{-i}}\right)\mathcal{C}_{-i,k}^{M} - \mathcal{C}_{i,k}^{D}\right]}^{\text{centrality trade-off}}}{\mathcal{E}_{-i,k}^{-i,k}-1}, \tag{42}$$

$$\frac{\tau_{ik}^{\mathcal{M}}-1}{\tau_{ik}^{\mathcal{M}}} = -\frac{\overbrace{\mathcal{T}_{ik}^{\mathcal{M}}}^{\text{ToT}} + \overbrace{\left[\left(\frac{M_i}{M_{-i}}\right)\mathcal{C}_{-i,i}^{M} - \mathcal{C}_{ik}^{D}\right]}^{\text{centrality trade-off}}}{\mathcal{E}_{ik}^{ik}-1}. \tag{43}$$

where $\mathcal{T}_{-i,k}^{\mathcal{X}}$ and $\mathcal{T}_{ik}^{\mathcal{M}}$ are terms-of-trade components, $\mathcal{E}_{ik}^{ik}$ and $\mathcal{E}_{-i,k}^{-i,k}$ are import demand elasticities. These terms-of-trade components can be expanded as

$$\mathcal{T}_{-i,k}^{\mathcal{X}} \equiv 1 + \left[\frac{F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}}}\right]^{-1}\left(\sum_{l\in\mathcal{K}_i\setminus\{k\}}\frac{\tau_{-i,l}^{\mathcal{X}}-1}{\tau_{-i,l}^{\mathcal{X}}\tau_{-i,l}^{\mathcal{M}}}F_{-i,l}\mathcal{E}_{-i,k}^{-i,l} + \sum_{l\in\mathcal{K}}\frac{\tau_{il}^{\mathcal{M}}-1}{\tau_{il}^{\mathcal{M}}}F_{il}\mathcal{E}_{-i,k}^{il}\right), \tag{44}$$

$$\mathcal{T}_{ik}^{\mathcal{M}} \equiv F_{ik}^{-1}\left(\sum_{l\in\mathcal{K}_i}\frac{(\tau_{-i,l}^{\mathcal{X}}-1)F_{-i,l}}{\tau_{-i,l}^{\mathcal{X}}\tau_{-i,l}^{\mathcal{M}}}\mathcal{E}_{ik}^{-i,l}+\sum_{l\in\mathcal{K}\setminus\{k\}}\frac{\tau_{il}^{\mathcal{M}}-1}{\tau_{il}^{\mathcal{M}}}F_{il}\mathcal{E}_{ik}^{il}\right), \tag{45}$$

where F_{jk} is the total cross-border flow from firm k to country j.

Proof. Following a small change in trade taxes, we rewrite the change in welfare as

$$dW_i = dR_i - C_i d\log P_i^C - M_i(d\log P_i^M - d\log P_{-i}^M). \tag{46}$$

There is a set of $\mathcal{K}$ firms in the global economy spanning two countries. There are $\mathcal{K}_i$ firms in the home economy and $\mathcal{K}_{-i}$ firms in the foreign economy.

The changes in firm prices are

$$d\log p_l = \sum_{k\in\mathcal{K}_i}\sum_{k'\in\mathcal{K}_{-i}}\Psi_{lk'}\Omega_{k'k}d\log\tau_{-i,k}^{\mathcal{X}}+\sum_{k'\in\mathcal{K}}\sum_{k\in\mathcal{K}_i}\Psi_{lk}\Omega_{kk'}d\log\tau_{ik'}^{\mathcal{M}} \tag{47}$$

The changes in aggregators' prices are

$$d\log P_i^C = \sum_{k\in\mathcal{K}_i}\left[\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}}s_{il}^C\Psi_{lk'}\Omega_{k'k}\right]d\log\tau_{-i,k}^{\mathcal{X}}+\sum_{k'\in\mathcal{K}}\left[s_{ik'}^C+\sum_{l\in\mathcal{K}}\sum_{k\in\mathcal{K}_i}s_{il}^C\Psi_{lk}\Omega_{kk'}\right]d\log\tau_{ik'}^{\mathcal{M}} \tag{48}$$

$$d\log P_i^M = \sum_{k\in\mathcal{K}_i}\left[\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}}s_{il}^M\Psi_{lk'}\Omega_{k'k}\right]d\log\tau_{-i,k}^{\mathcal{X}}+\sum_{k'\in\mathcal{K}}\left[s_{ik'}^M+\sum_{l\in\mathcal{K}}\sum_{k\in\mathcal{K}_i}s_{il}^M\Psi_{lk}\Omega_{kk'}\right]d\log\tau_{ik'}^{\mathcal{M}} \tag{49}$$

$$d\log P_{-i}^M = \sum_{k\in\mathcal{K}_i}\left[s_{-i,k}^M+\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}}s_{-i,l}^M\Psi_{lk'}\Omega_{k'k}\right]d\log\tau_{-i,k}^{\mathcal{X}}+\sum_{k'\in\mathcal{K}}\left[\sum_{l\in\mathcal{K}}\sum_{k\in\mathcal{K}_i}s_{-i,l}^M\Psi_{lk}\Omega_{kk'}\right]d\log\tau_{ik'}^{\mathcal{M}} \tag{50}$$

The changes in revenues are

$$dR_i = \sum_{k\in\mathcal{K}_i}\frac{F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}}\tau_{-i,k}^{\mathcal{X}}}d\log\tau_{-i,k}^{\mathcal{X}}+\sum_{k\in\mathcal{K}_i}\frac{(\tau_{-i,k}^{\mathcal{X}}-1)F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}}\tau_{-i,k}^{\mathcal{X}}}d\log F_{-i,k}$$
$$+\sum_{k'\in\mathcal{K}}\frac{F_{ik'}}{\tau_{ik'}^{\mathcal{M}}}d\log\tau_{ik'}^{\mathcal{M}}+\sum_{k\in\mathcal{K}}\frac{(\tau_{ik}^{\mathcal{M}}-1)F_{ik}}{\tau_{ik}^{\mathcal{M}}}d\log F_{ik}, \tag{51}$$

where F_{ik} denotes the aggregate flow from firm k into country i and is characterized by the elasticity of demand $\mathcal{E}_{ol'}^{jk'} \equiv d\log F_{jk'}/d\log\tau_{ol'}$.

The elasticities can be further decomposed. One can write

$$d\log X_l = \sum_{k'\in\mathcal{K}}\sum_{j\in\{H,F\}}\frac{\tilde{\Psi}_{k'l}\tilde{E}_{jk'}}{X_l}d\log\tilde{E}_{jk'}-\sum_{k\in\mathcal{K}_i}\sum_{k'\in\mathcal{K}_{-i}}\frac{\tilde{\Psi}_{k'l}\tilde{\Omega}_{k'k}X_{k'}}{X_l}d\log\tau_{-i,k}^{\mathcal{X}} \tag{52}$$

$$-\sum_{k'\in\mathcal{K}}\sum_{k\in\mathcal{K}_i}\frac{\tilde{\Psi}_{lk}\tilde{\Omega}_{kk'}X_k}{X_l}d\log\tau_{ik'}^{\mathcal{M}}. \tag{53}$$

We express $\mathtt{E}_{ol}^{jk} = d\log E_{jk}/d\log\tau_{ol}$ as

$$\begin{aligned}\mathtt{E}_{ol'}^{jk'} = S_{jk'}^{C}(1-\eta_j)\left[\mathbf{1}\{j=o\}s_{ol'}^{C} + \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{K}_o} s_{jk}^{C}\Psi_{kl}\Omega_{ll'}\right] \\ + \mathbf{1}\{j=i\}S_{jk'}^{M}(1-\zeta_j)\left[\mathbf{1}\{j=o\}s_{ol'}^{M} + \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{K}_o} s_{ik}^{M}\Psi_{kl}\Omega_{ll'}\right] d\log\tau_{ol}\end{aligned} \tag{54}$$

Then we express $\mathcal{E}_{ol'}^{jk'} = d\log F_{jk'}/d\log\tau_{ol'}$ as

$$\mathcal{E}_{ol'}^{jk'} = \mathcal{S}_{jk'}^{E}\mathtt{E}_{ol'}^{jk'} + \sum_{k\in\mathcal{K}_j}\mathcal{S}_{kk'}^{\Omega}\left[\sum_{j'\in\{H,F\}}\sum_{l\in\mathcal{K}}\frac{\tilde{\Psi}_{lk}\tilde{E}_{j'l}}{X_k}\tilde{\mathcal{E}}_{ol'}^{j'l} - \sum_{l\in\mathcal{K}_o}\frac{\tilde{\Psi}_{l'k}\tilde{\Omega}_{ll'}X_l}{X_k}\right]. \tag{55}$$

The changes in revenues can thus be recast as

$$\begin{aligned}dR_i = \sum_{k\in\mathcal{K}_i}\left\{\frac{F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}}\tau_{-i,k}^{\mathcal{X}}} + \sum_{l\in\mathcal{K}_i}\frac{(\tau_{-i,l}^{\mathcal{X}}-1)F_{-i,l}}{\tau_{-i,l}^{\mathcal{M}}\tau_{-i,l}^{\mathcal{X}}}\mathcal{E}_{-i,k}^{-i,l} + \sum_{l\in\mathcal{K}}\frac{\tau_{il}^{\mathcal{M}}-1}{\tau_{il}^{\mathcal{M}}}F_{il}\mathcal{E}_{-i,k}^{il}\right\} d\log\tau_{-i,k}^{\mathcal{X}} \\ + \sum_{k'\in\mathcal{K}}\left\{\frac{F_{ik'}}{\tau_{ik'}^{\mathcal{M}}} + \sum_{l\in\mathcal{K}_i}\frac{(\tau_{-i,l}^{\mathcal{X}}-1)F_{-i,l}}{\tau_{-i,l}^{\mathcal{M}}\tau_{-i,l}^{\mathcal{X}}}\mathcal{E}_{ik'}^{-i,l} + \sum_{l\in\mathcal{K}}\frac{\tau_{il}^{\mathcal{M}}-1}{\tau_{il}^{\mathcal{M}}}F_{il}\mathcal{E}_{ik'}^{il}\right\} d\log\tau_{ik'}^{\mathcal{M}}\end{aligned} \tag{56}$$

or

$$\begin{aligned}dR_i = \sum_{k\in\mathcal{K}_i}\Bigg\{\frac{F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}}} + \frac{(\tau_{-i,k}^{\mathcal{X}}-1)F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}}\tau_{-i,k}^{\mathcal{X}}}(\mathcal{E}_{-i,k}^{-i,k}-1) \\ + \sum_{l\in\mathcal{K}_i\setminus\{k\}}\frac{(\tau_{-i,l}^{\mathcal{X}}-1)F_{-i,l}}{\tau_{-i,l}^{\mathcal{M}}\tau_{-i,l}^{\mathcal{X}}}\mathcal{E}_{-i,k}^{-i,l} + \sum_{l\in\mathcal{K}}\frac{\tau_{il}^{\mathcal{M}}-1}{\tau_{il}^{\mathcal{M}}}F_{il}\mathcal{E}_{-i,k}^{il}\Bigg\} d\log\tau_{-i,k}^{\mathcal{X}} \\ + \sum_{k'\in\mathcal{K}}\Bigg\{F_{ik'} + \frac{\tau_{ik'}^{\mathcal{M}}-1}{\tau_{ik'}^{\mathcal{M}}}F_{ik'}(\mathcal{E}_{ik'}^{ik'}-1) + \\ + \sum_{l\in\mathcal{K}_i}\frac{(\tau_{-i,l}^{\mathcal{X}}-1)F_{-i,l}}{\tau_{-i,l}^{\mathcal{M}}\tau_{-i,l}^{\mathcal{X}}}\mathcal{E}_{ik'}^{-i,l} + \sum_{l\in\mathcal{K}\setminus\{k\}}\frac{\tau_{il}^{\mathcal{M}}-1}{\tau_{il}^{\mathcal{M}}}F_{il}\mathcal{E}_{ik'}^{il}\Bigg\} d\log\tau_{ik'}^{\mathcal{M}}\end{aligned} \tag{57}$$

Collecting revenue changes and price changes into welfare changes yields

$$
\begin{aligned}
dW_i = \sum_{k\in\mathcal{K}_i} \Bigg\{ & \frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}} + \frac{(\tau^{\mathcal{X}}_{-i,k}-1)F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}\tau^{\mathcal{X}}_{-i,k}}(\mathcal{E}^{-i,k}_{-i,k}-1) \\
& + \sum_{l\in\mathcal{K}_i\setminus\{k\}} \frac{(\tau^{\mathcal{X}}_{-i,l}-1)F_{-i,l}}{\tau^{\mathcal{M}}_{-i,l}\tau^{\mathcal{X}}_{-i,l}}\mathcal{E}^{-i,l}_{-i,k} + \sum_{l\in\mathcal{K}} \frac{\tau^{\mathcal{M}}_{il}-1}{\tau^{\mathcal{M}}_{il}} F_{il}\mathcal{E}^{il}_{-i,k} \\
& - C_i \left[\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}} s^C_{il}\Psi_{lk'}\Omega_{k'k}\right] - M_i \left[\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}} s^M_{il}\Psi_{lk'}\Omega_{k'k}\right] \\
& + M_i \left[s^M_{-i,k} + \sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}} s^M_{-i,l}\Psi_{lk'}\Omega_{k'k}\right]\Bigg\} d\log\tau^{\mathcal{X}}_{-i,k} \\
+ \sum_{k'\in\mathcal{K}} \Bigg\{ & F_{ik'} + \frac{\tau^{\mathcal{M}}_{ik'}-1}{\tau^{\mathcal{M}}_{ik'}} F_{ik'}(\mathcal{E}^{ik'}_{ik'}-1) + \\
& + \sum_{l\in\mathcal{K}_i} \frac{(\tau^{\mathcal{X}}_{-i,l}-1)F_{-i,l}}{\tau^{\mathcal{M}}_{-i,l}\tau^{\mathcal{X}}_{-i,l}}\mathcal{E}^{-i,l}_{ik'} + \sum_{l\in\mathcal{K}\setminus\{k\}} \frac{\tau^{\mathcal{M}}_{il}-1}{\tau^{\mathcal{M}}_{il}} F_{il}\mathcal{E}^{il}_{ik'} \\
& - C_i \left[s^C_{ik'} + \sum_{l\in\mathcal{K}}\sum_{k\in\mathcal{K}_i} s^C_{il}\Psi_{lk}\Omega_{kk'}\right] - M_i \left[s^M_{ik'} + \sum_{l\in\mathcal{K}}\sum_{k\in\mathcal{K}_i} s^M_{il}\Psi_{lk}\Omega_{kk'}\right] \\
& + M_i \left[\sum_{l\in\mathcal{K}}\sum_{k\in\mathcal{K}_i} s^M_{-i,l}\Psi_{lk}\Omega_{kk'}\right]\Bigg\} d\log\tau^{\mathcal{M}}_{ik'}
\end{aligned}
\tag{58}
$$

One can take the first term in isolation for a given k and rewrite it as

$$\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\left\{\frac{\tau^{\mathcal{X}}_{-i,k}-1}{\tau^{\mathcal{X}}_{-i,k}}(\mathcal{E}^{-i,k}_{-i,k}-1)+1+\left[\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\right]^{-1}\left(\sum_{l\in\mathcal{K}_i\setminus\{k\}}R_{-i,l}\mathcal{E}^{-i,l}_{-i,k}+\sum_{l\in\mathcal{K}}R_{il}\mathcal{E}^{il}_{-i,k}\right)\right. \tag{59}$$

$$+\tau^{\mathcal{M}}_{-i,k}\left(\frac{M_i}{M_{-i}}\right)\left[\frac{E_{-i,k}}{F_{-i,k}}\frac{s^M_{-i,k}M_{-i}}{E_{-i,k}}+\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}}\frac{X_{k'}\Omega_{k'k}}{F_{-i,k}}\frac{s^M_{-i,l}M_{-i}}{E_{-i,l}}\frac{\tilde{E}_{-i,l}\tilde{\Psi}_{lk'}}{X_{k'}}\frac{\tau_{-i,l}\Psi_{lk'}}{\tilde{\Psi}_{lk'}}\right]$$

$$\left.-\sum_{l\in\mathcal{K}}\sum_{k'\in\mathcal{K}_{-i}}\frac{X_{k'}\Omega_{k'k}}{F_{-i,k}}\frac{\tilde{E}_{i,l}\tilde{\Psi}_{lk'}}{X_{k'}}\frac{\tau_{il}\Psi_{lk'}}{\tilde{\Psi}_{lk'}}\right\}$$

$$=\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\left\{\frac{\tau^{\mathcal{X}}_{-i,k}-1}{\tau^{\mathcal{X}}_{-i,k}}(\mathcal{E}^{-i,k}_{-i,k}-1)+\mathcal{T}^{\mathcal{X}}_{-i,k}\right.$$

$$+\tau^{\mathcal{M}}_{-i,k}\left(\frac{M_i}{M_{-i}}\right)\left[S^E_{-i,k}S^M_{-i,k}+\sum_{k'\in\mathcal{K}_{-i}}S^\Omega_{k'k}\sum_{l\in\mathcal{K}}S^M_{-i,l}\omega^{(-i)}_{lk'}\beta^{(-i)}_{lk'}\right]$$

$$\left.-\tau^{\mathcal{M}}_{-i,k}\sum_{k'\in\mathcal{K}_{-i}}S^\Omega_{k'k}\sum_{l\in\mathcal{K}}\omega^{(i)}_{lk'}\beta^{(i)}_{lk'}\right\}$$

$$=\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\left\{\frac{\tau^{\mathcal{X}}_{-i,k}-1}{\tau^{\mathcal{X}}_{-i,k}}(\mathcal{E}^{-i,k}_{-i,k}-1)+\mathcal{T}^{\mathcal{X}}_{-i,k}+\tau^{\mathcal{M}}_{-i,k}\left(\frac{M_i}{M_{-i}}\right)\left[S^E_{-i,k}S^M_{-i,k}+\sum_{k'\in\mathcal{K}_{-i}}S^\Omega_{k'k}\mathcal{C}^M_{-i,k'}\right]\right.$$

$$\left.-\tau^{\mathcal{M}}_{-i,k}\sum_{k'\in\mathcal{K}_{-i}}S^\Omega_{k'k}\mathcal{C}^D_{ik'}\right\}$$

Analogously, the second term can be rewritten as

$$F_{ik'}\left\{\frac{\tau^{\mathcal{M}}_{ik'}-1}{\tau^{\mathcal{M}}_{ik'}}(\mathcal{E}^{ik'}_{ik'}-1)+\mathcal{T}^{\mathcal{M}}_{ik'}+\left(\frac{M_i}{M_{-i}}\right)\sum_{k\in\mathcal{K}_i}S^\Omega_{kk'}\mathcal{C}^M_{-i,k}-S^E_{ik'}-\sum_{k\in\mathcal{K}_i}S^\Omega_{kk'}C^D_{ik}\right\} \tag{60}$$

That makes optimal taxes

$$\frac{\tau^{\mathcal{X}}_{-i,k}-1}{\tau^{\mathcal{X}}_{-i,k}}=\frac{\mathcal{T}^{\mathcal{X}}_{-i,k}+\tau^{\mathcal{M}}_{-i,k}\left\{\left(\frac{M_i}{M_{-i}}\right)\left[S^E_{-i,k}S^M_{-i,k}+\sum_{k'\in\mathcal{K}_{-i}}S^\Omega_{k'k}\mathcal{C}^M_{-i,k'}\right]-\sum_{k'\in\mathcal{K}_{-i}}S^\Omega_{k'k}\mathcal{C}^D_{ik'}\right\}}{\mathcal{E}^{-i,k}_{-i,k}-1}, \tag{61}$$

$$\frac{\tau^{\mathcal{M}}_{ik'}-1}{\tau^{\mathcal{M}}_{ik'}}=\frac{\mathcal{T}^{\mathcal{M}}_{ik'}+\left\{\left(\frac{M_i}{M_{-i}}\right)\sum_{k\in\mathcal{K}_i}S^\Omega_{kk'}\mathcal{C}^M_{-i,k}-\left[S^E_{ik'}+\sum_{k\in\mathcal{K}_i}S^\Omega_{kk'}C^D_{ik}\right]\right\}}{\mathcal{E}^{ik'}_{ik'}-1} \tag{62}$$

Without loss of generality, assume that for the taxed nodes there is only one connecting node in a given country that takes all the sales and also has no final demand from the aggregator. For instance, for an exporting firm k in country i that would be an importing mirror node in country $-i$ with no final demand from aggregators in country $-i$. This would simplify the tax formulas to

$$\frac{\tau^{\mathcal{X}}_{-i,k}-1}{\tau^{\mathcal{X}}_{-i,k}} = \frac{\mathcal{T}^{\mathcal{X}}_{-i,k} + \tau^{\mathcal{M}}_{-i,k}\left[\left(\frac{M_i}{M_{-i}}\right)\mathcal{C}^{M}_{-i,k} - \mathcal{C}^{D}_{-i,k}\right]}{\mathcal{E}^{-i,k}_{-i,k}-1}, \tag{63}$$

$$\frac{\tau^{\mathcal{M}}_{ik}-1}{\tau^{\mathcal{M}}_{ik}} = \frac{\mathcal{T}^{\mathcal{M}}_{ik} + \left[\left(\frac{M_i}{M_{-i}}\right)\mathcal{C}^{M}_{-i,k} - C^{D}_{ik}\right]}{\mathcal{E}^{ik}_{ik}-1}, \tag{64}$$

which is exactly the proposition formula. □

Proposition B.3 subsumes Proposition B.1. There are two main changes in the tax formulas compared to Proposition B.1. First, the final sales share S^M is replaced by the military centrality $\mathcal{C}^M$. In the horizontal case, one can verify that the military centrality equals the sales share exactly. Second, there is the addition of a new distortion centrality term $\mathcal{C}^D$. This term captures the impact of roundabout imports. In the horizontal case with no roundabout component, distortion centrality equals zero, as exported goods never return as re-imports into the domestic economy.

C Empirical measurement

C.1 EU dual-use list

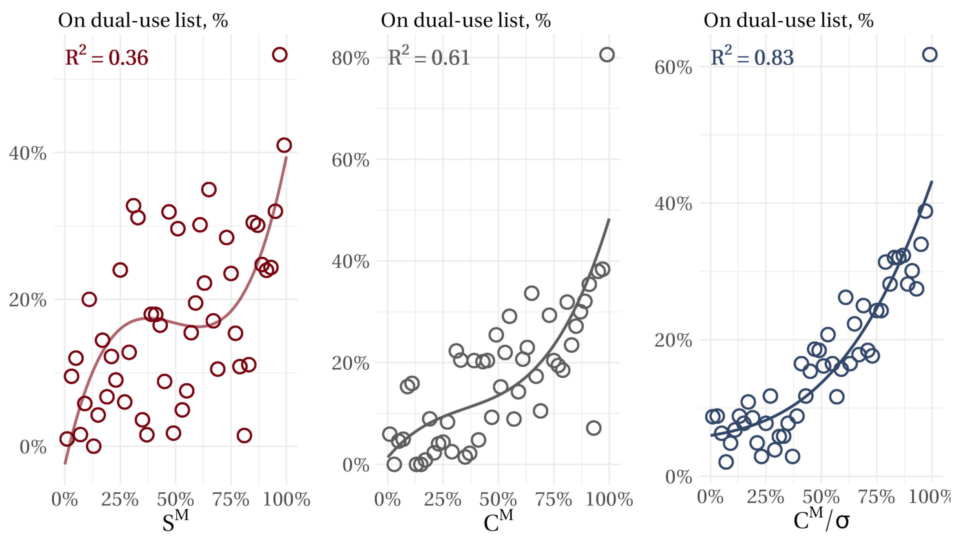


Figure OA.C.1: Robustness: Includes military subcontracts

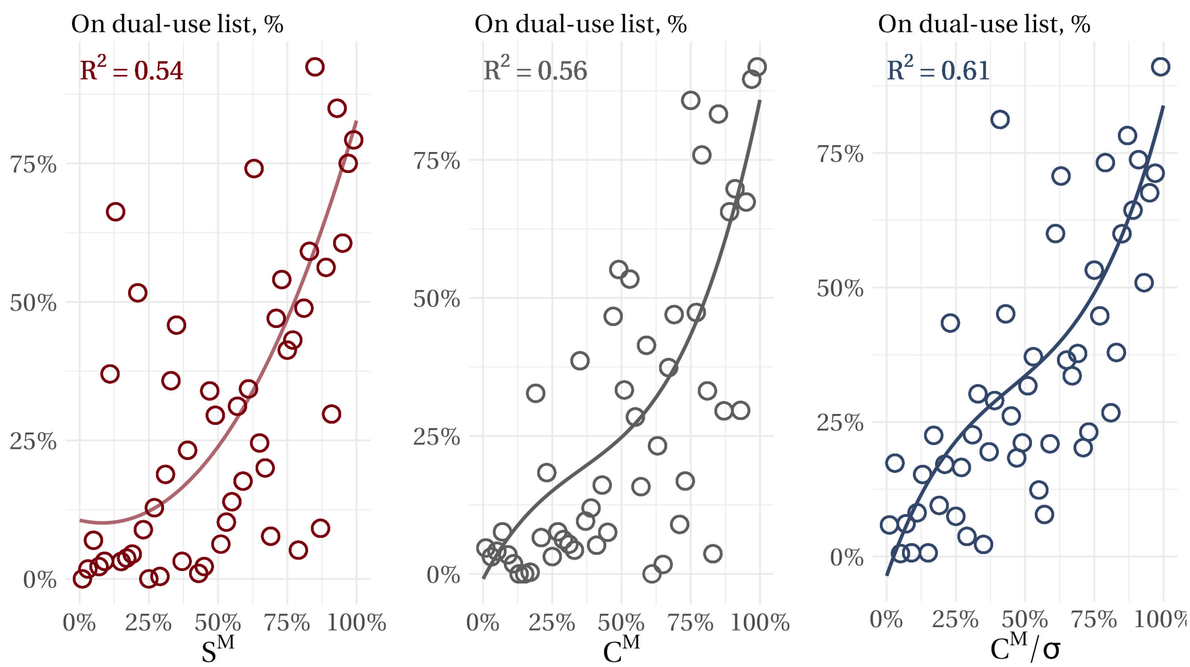


Figure OA.C.2: Robustness: Weighted by trade flows

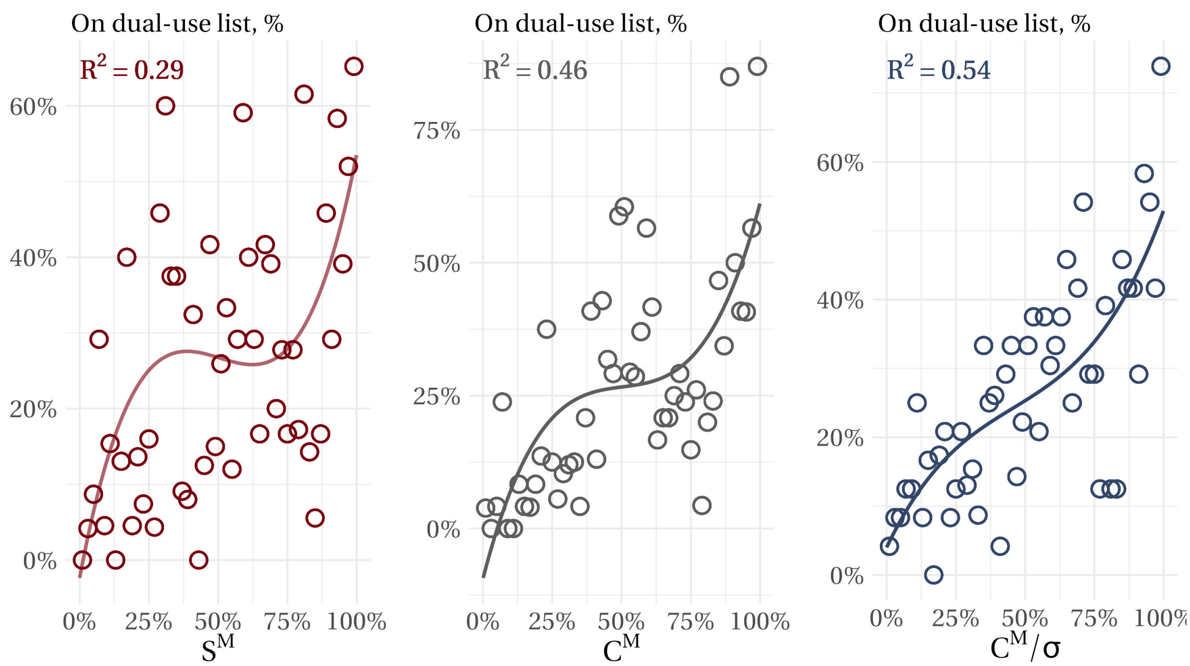


Figure OA.C.3: Robustness: 4-digit level

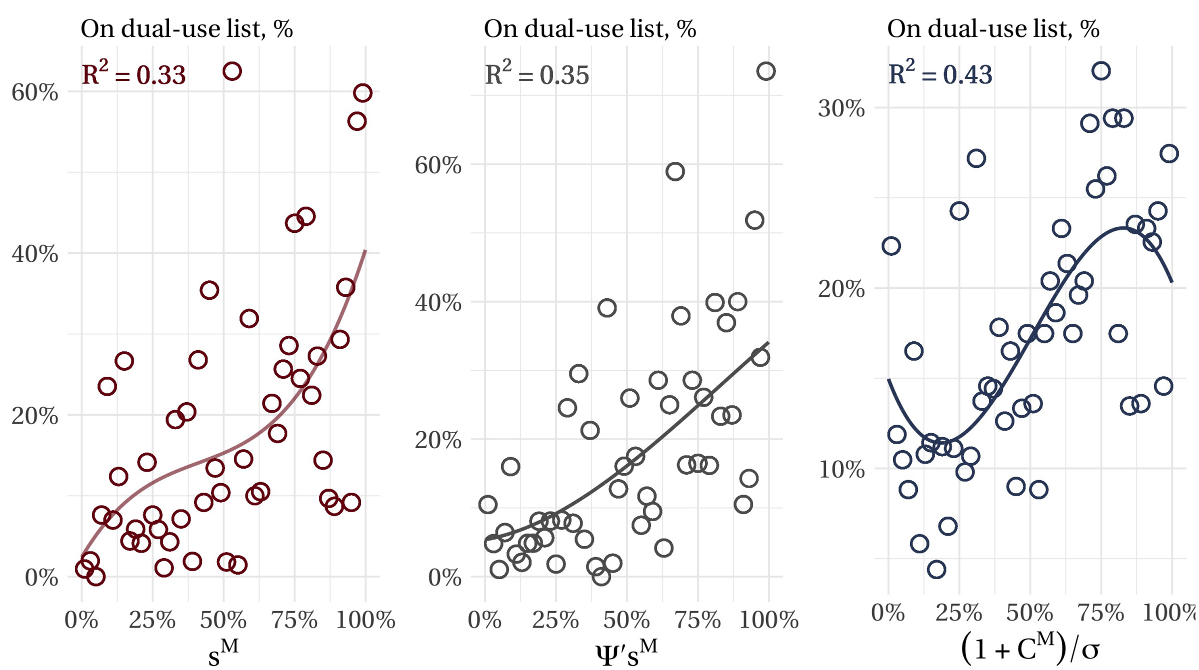


Figure OA.C.4: Alternative measures

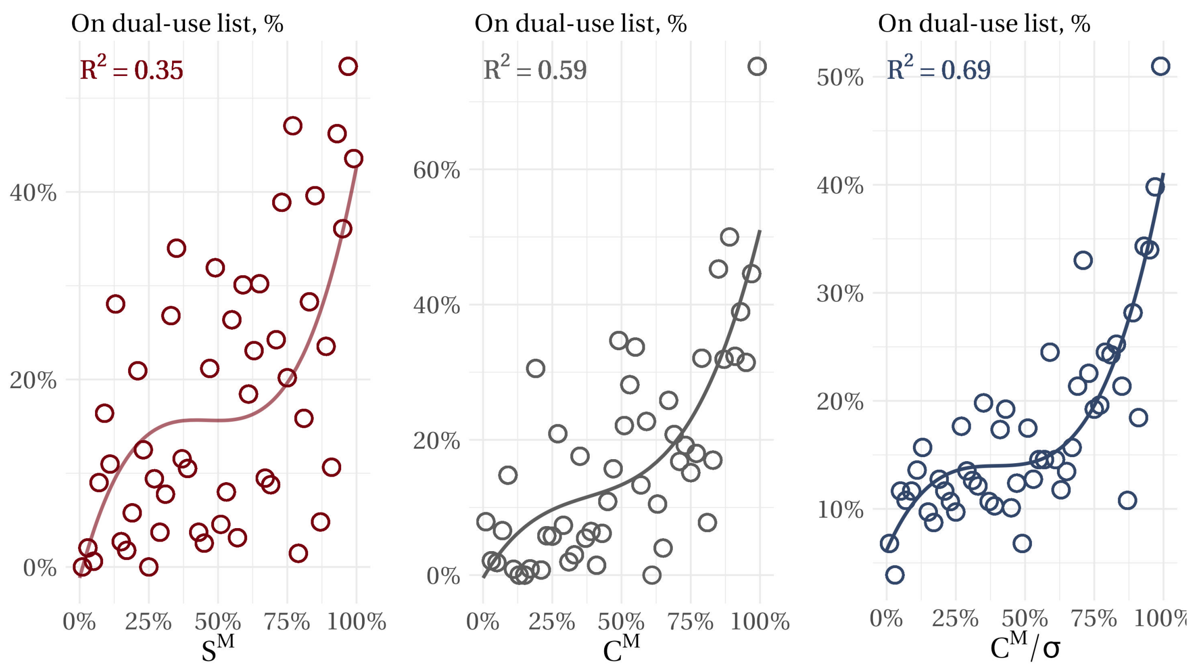


Figure OA.C.5: Robustness: Soderbery (2015) based on Broda and Weinstein (2006)

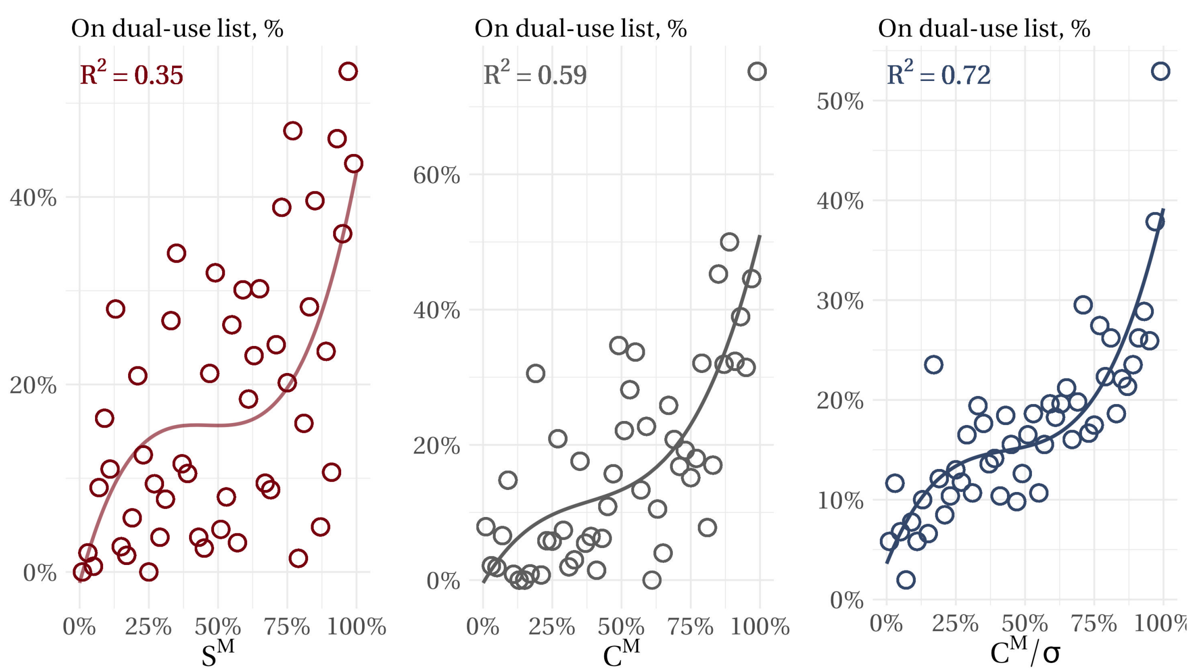


Figure OA.C.6: Robustness: Fontagné et al. (2022) elasticities

C.2 The U.S. Export NTMs after 2022, figures

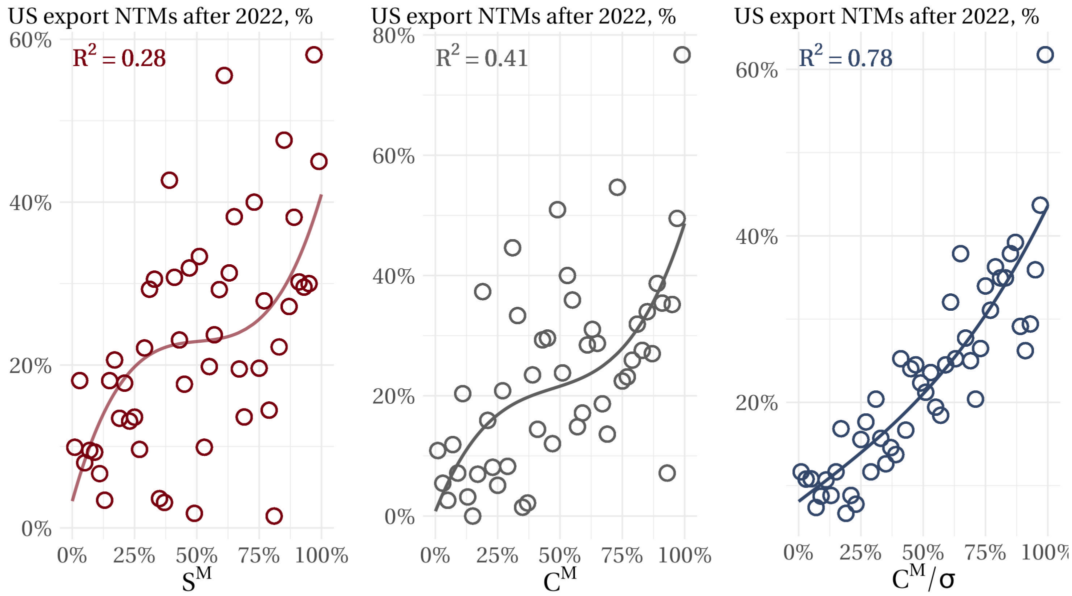


Figure OA.C.7: Robustness: Includes military subcontracts

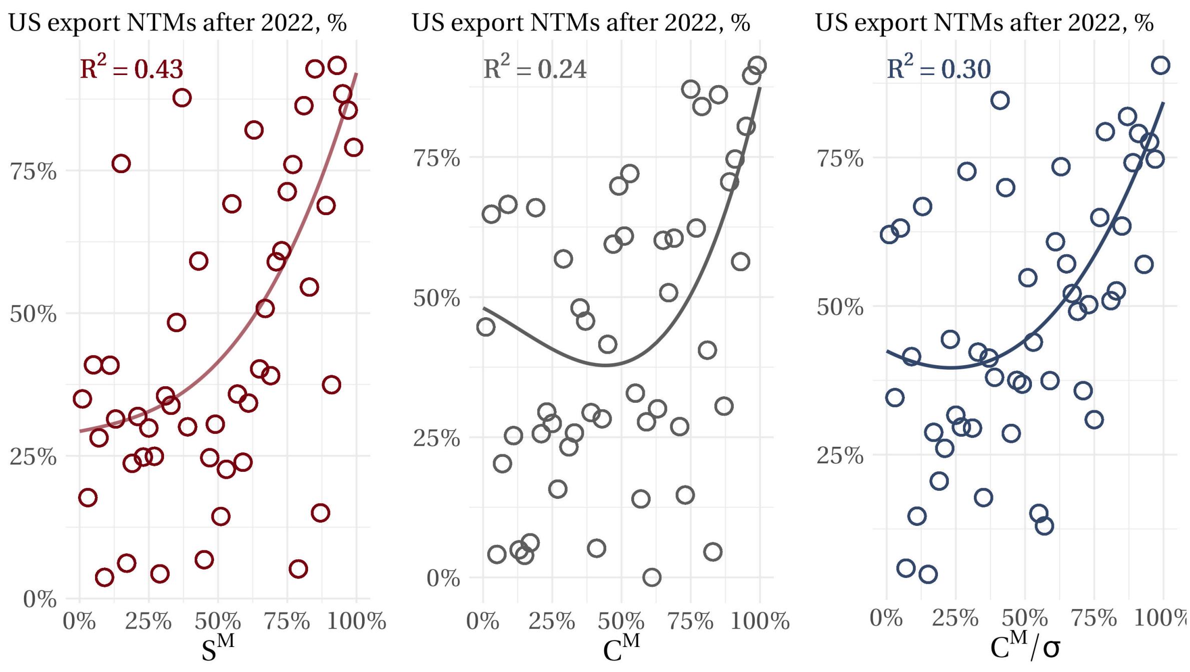


Figure OA.C.8: Robustness: Weighted by trade flows

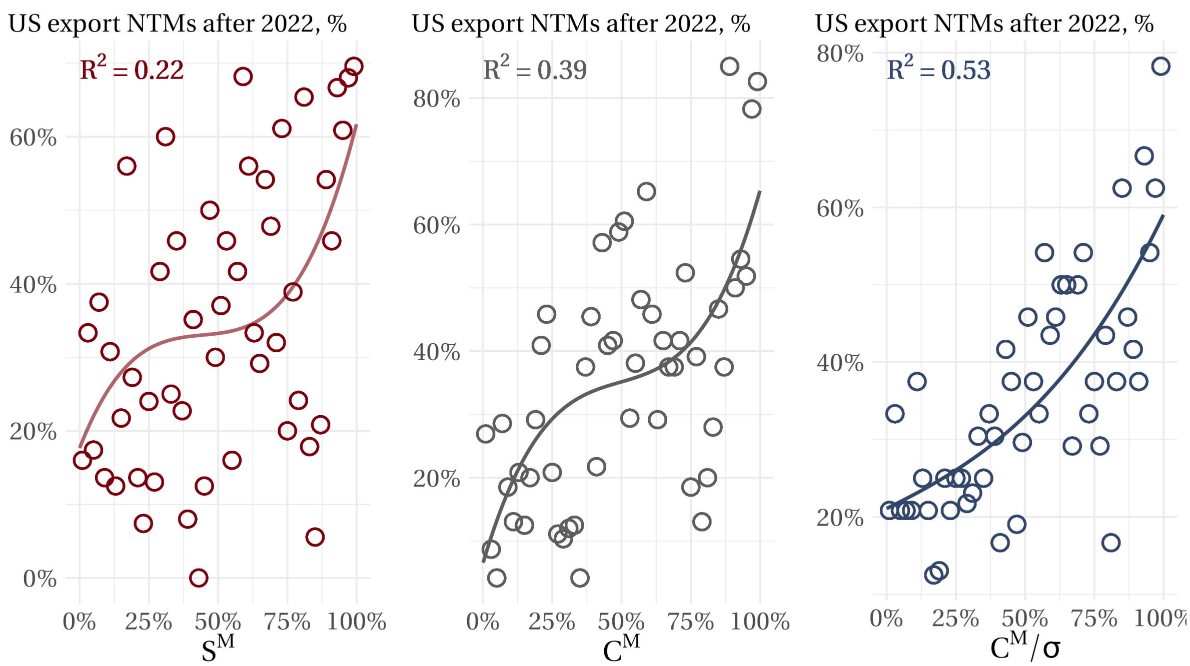


Figure OA.C.9: Robustness: 4-digit codes

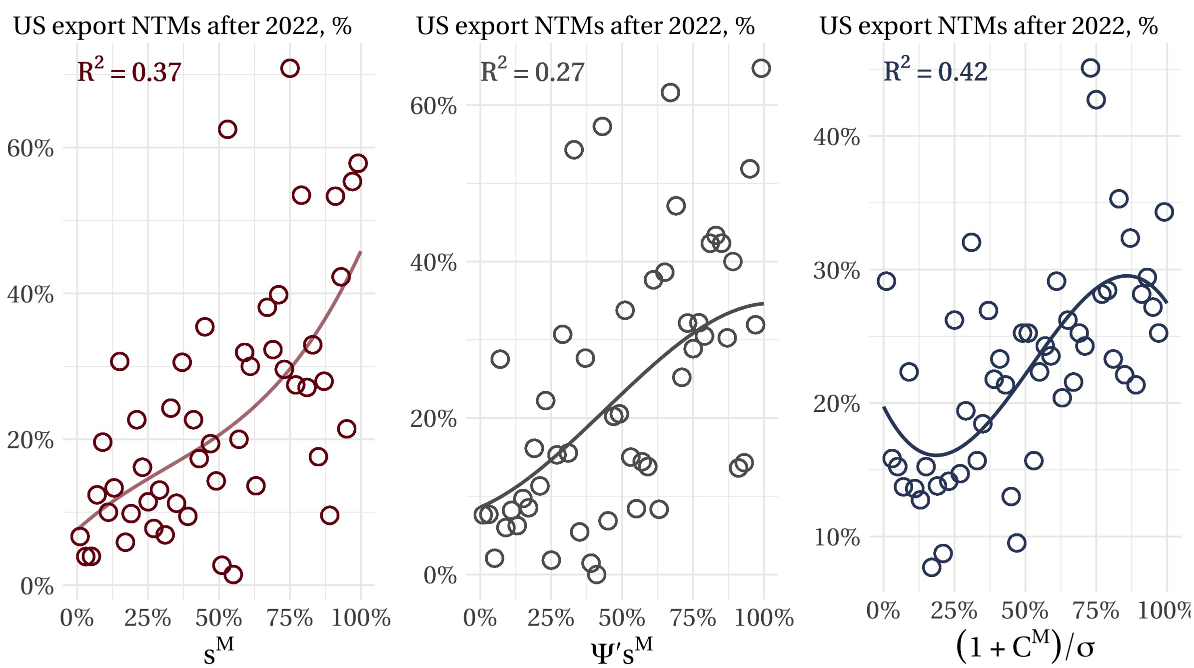


Figure OA.C.10: Robustness: Alternative measures

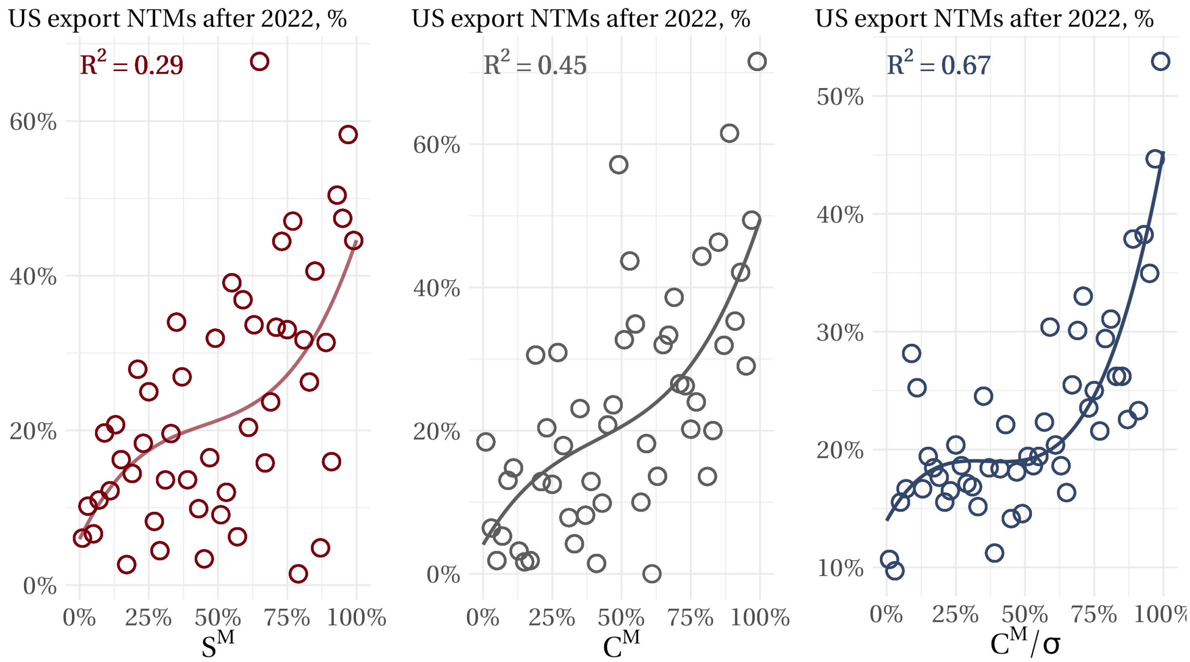


Figure OA.C.11: Robustness: Soderbery (2015) based on Broda and Weinstein (2006)

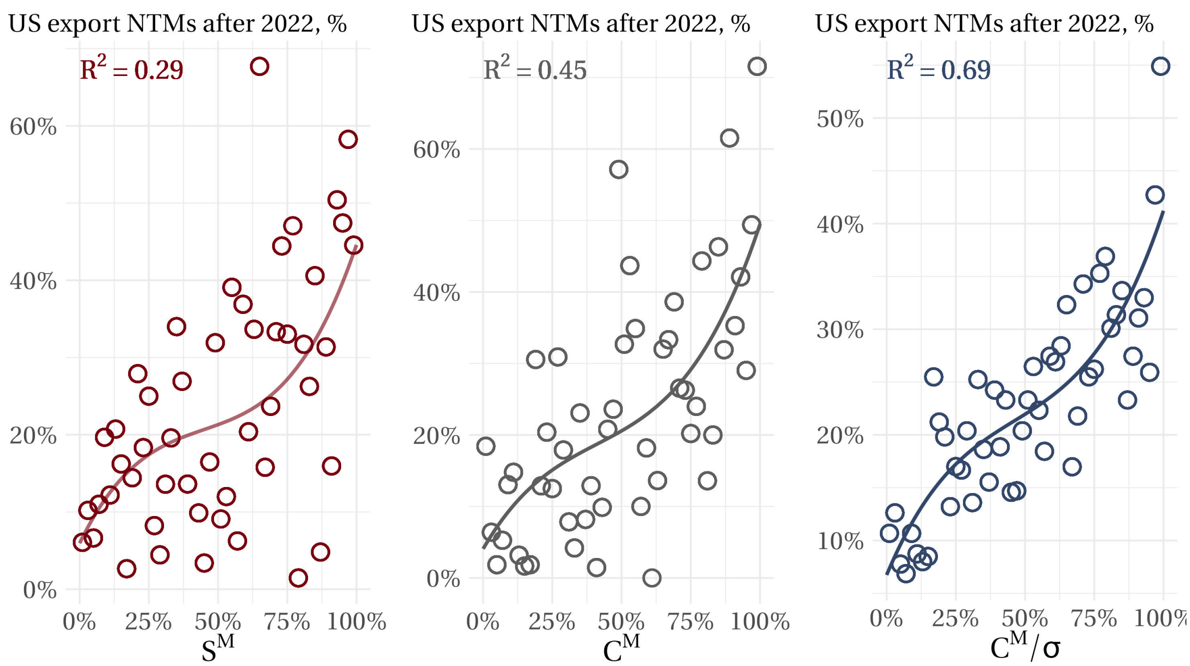


Figure OA.C.12: Robustness: Fontagné et al. (2022) elasticities

C.3 The U.S. NTMs after 2022, tables

Dependent Variable:	Had a US export NTM after 2022						
Model:	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Variables							
S_{US}^{M}	0.5773***		0.0783				
	(0.0926)		(0.0915)				
$\mathcal{C}_{\text{US}}^{M}/\sigma$		2.589***	2.443***	2.179***	2.166***	1.947***	0.7805*
		(0.3339)	(0.3825)	(0.3200)	(0.3249)	(0.2984)	(0.3037)
Fixed-effects							
Polynomial S_{US}^{M}				Yes			
Piecewise S_{US}^{M}					Yes	Yes	Yes
Goods controls (trade, sales, ...)						Yes	Yes
HS 2-digit							Yes
Fit statistics							
Observations	5,135	5,135	5,135	5,135	5,135	5,134	5,134
R^2	0.01597	0.03529	0.03547	0.05602	0.06666	0.16382	0.38737
Within R^2				0.05602	0.02950	0.03845	0.01941

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Table OA.C.1: The U.S. NTMs after 2022: Military sales share versus military use

Dependent Variable:	Had a US export NTM after 2022							
Model:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Variables								
$\mathcal{C}_{\text{US}}^{M}/\sigma$	1.520***	2.109***	1.938***	1.993***				
	(0.2960)	(0.3018)	(0.2919)	(0.2748)				
rank $\mathcal{C}_{\text{US}}^{M}/\sigma$					0.2908***	0.3275***	0.2674***	0.2859***
					(0.0208)	(0.0231)	(0.0210)	(0.0256)
Fixed-effects								
Polynomial s_{US}^{M}	Yes				Yes			
Polynomial $\Psi' s_{\text{US}}^{M}$		Yes				Yes		
Polynomial rank s_{US}^{M}			Yes				Yes	
Polynomial rank $\Psi' s_{\text{US}}^{M}$				Yes				Yes
Fit statistics								
Observations	5,135	5,135	5,135	5,135	5,135	5,135	5,135	5,135
R^2	0.05141	0.04688	0.07284	0.06029	0.07901	0.06444	0.08586	0.06546
Within R^2	0.05141	0.04688	0.07284	0.06029	0.07901	0.06444	0.08586	0.06546

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Table OA.C.2: The U.S. NTMs after 2022: Military use and rank of military use

D Calibration

D.1 Proof of Proposition 4

The welfare is

$$W_i = U_i(\{c_j\}, \{m_j\}) \tag{65}$$

subject to $C_i = w_i L_i + R_i + D_i - M_i$.

Following a small change in trade taxes, we rewrite the change in welfare as

$$\begin{aligned} dW_i &= \sum_j U_{ic,j} dc_j + U_{im,j} dm_j \\ &= \sum_j \frac{U_{ic,j}}{P_j^C} dR_j + U_{ic,j} \frac{w_j L_j}{P_j^C} d\log w_j - U_{ic,j} \frac{C_j}{P_j^C} d\log P_j^C - U_{im,j} \frac{M_j}{P_j^M} d\log P_j^M \end{aligned} \tag{66}$$

Following a tax intervention, the changes in firm prices are

$$\begin{aligned} d\log p_l = &\sum_{k\in\mathcal{K}_i} \sum_{k'\in\mathcal{K}_{-i}} \Psi_{lk'} \Omega_{k'k} d\log \tau_{-i,k}^{\mathcal{X}} + \sum_{k'\in\mathcal{K}} \sum_{k\in\mathcal{K}_i} \Psi_{lk} \Omega_{kk'} d\log \tau_{ik'}^{\mathcal{M}} \\ &+ \sum_{k'\in\mathcal{K}} \sum_{j'} \Psi_{lk'} \Omega_{k'j'}^L d\log w_{j'} \end{aligned} \tag{67}$$

The changes in aggregators' $A \in \{C, M\}$ prices are

$$\begin{aligned} d\log P_{-i}^A = &\sum_{k\in\mathcal{K}_i} \left[s_{-i,k}^A + \sum_{l\in\mathcal{K}} \sum_{k'\in\mathcal{K}_{-i}} s_{-il}^A \Psi_{lk'} \Omega_{k'k} \right] d\log \tau_{-i,k}^{\mathcal{X}} \\ &+ \sum_{k'\in\mathcal{K}} \left[\sum_{l\in\mathcal{K}} \sum_{k\in\mathcal{K}_i} s_{-il}^A \Psi_{lk} \Omega_{kk'} \right] d\log \tau_{ik'}^{\mathcal{M}} + \sum_{k'\in\mathcal{K}} \sum_{l,j'} s_{-il}^A \Psi_{lk'} \Omega_{k'j'}^L d\log w_{j'} \end{aligned} \tag{68}$$

$$\begin{aligned} d\log P_i^A = &\sum_{k\in\mathcal{K}_i} \left[\sum_{l\in\mathcal{K}} \sum_{k'\in\mathcal{K}_{-i}} s_{il}^A \Psi_{lk'} \Omega_{k'k} \right] d\log \tau_{-i,k}^{\mathcal{X}} \\ &+ \sum_{k'\in\mathcal{K}} \left[s_{ik'}^A + \sum_{l\in\mathcal{K}} \sum_{k\in\mathcal{K}_i} s_{il}^A \Psi_{lk} \Omega_{kk'} \right] d\log \tau_{ik'}^{\mathcal{M}} + \sum_{k'\in\mathcal{K}} \sum_{l,j'} s_{il}^A \Psi_{lk'} \Omega_{k'j'}^L d\log w_{j'} \end{aligned} \tag{69}$$

The changes in revenues are

$$\begin{aligned} dR_i = &\sum_{k\in\mathcal{K}_i} \frac{F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}} \tau_{-i,k}^{\mathcal{X}}} d\log \tau_{-i,k}^{\mathcal{X}} + \sum_{k\in\mathcal{K}_i} \frac{(\tau_{-i,k}^{\mathcal{X}} - 1) F_{-i,k}}{\tau_{-i,k}^{\mathcal{M}} \tau_{-i,k}^{\mathcal{X}}} d\log F_{-i,k} \\ &+ \sum_{k'\in\mathcal{K}} \frac{F_{ik'}}{\tau_{ik'}^{\mathcal{M}}} d\log \tau_{ik'}^{\mathcal{M}} + \sum_{k\in\mathcal{K}} \frac{(\tau_{ik}^{\mathcal{M}} - 1) F_{ik}}{\tau_{ik}^{\mathcal{M}}} d\log F_{ik}, \end{aligned} \tag{70}$$

$$dR_{-i} = \sum_{k\in\mathcal{K}} \frac{(\tau^{\mathcal{M}}_{-i,k}-1)F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}} d\log F_{-i,k} + \sum_{k\notin\mathcal{K}_{-i}} \frac{(\tau^{\mathcal{X}}_{-i,k}-1)F_{-i,k}}{\tau^{\mathcal{X}}_{-i,k}\tau^{\mathcal{M}}_{-i,k}} d\log F_{-i,k}, \tag{71}$$

where F_{ik} denotes the aggregate flow from firm k into country i and is characterized by the macro-elasticity of demand $\mathcal{E}^{jk'}_{ol'} \equiv d\log F_{jk'}/d\log \tau_{ol'}$.

The changes in home revenues can thus be recast as

$$dR_i = \sum_{k\in\mathcal{K}_i} \left\{ \frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}} + \frac{(\tau^{\mathcal{X}}_{-i,k}-1)F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}\tau^{\mathcal{X}}_{-i,k}} (\mathcal{E}^{-i,k}_{-i,k} - 1) \right. \tag{72}$$

$$\left. + \sum_{l\in\mathcal{K}_i\backslash\{k\}} \frac{(\tau^{\mathcal{X}}_{-i,l}-1)F_{-i,l}}{\tau^{\mathcal{M}}_{-i,l}\tau^{\mathcal{X}}_{-i,l}} \mathcal{E}^{-i,l}_{-i,k} + \sum_{l\in\mathcal{K}} \frac{\tau^{\mathcal{M}}_{il}-1}{\tau^{\mathcal{M}}_{il}} F_{il}\mathcal{E}^{il}_{-i,k} \right\} d\log \tau^{\mathcal{X}}_{-i,k} \tag{73}$$

$$+ \sum_{k'\in\mathcal{K}} \left\{ F_{ik'} + \frac{\tau^{\mathcal{M}}_{ik'}-1}{\tau^{\mathcal{M}}_{ik'}} F_{ik'}(\mathcal{E}^{ik'}_{ik'} - 1) + \right.$$

$$\left. + \sum_{l\in\mathcal{K}_i} \frac{(\tau^{\mathcal{X}}_{-i,l}-1)F_{-i,l}}{\tau^{\mathcal{M}}_{-i,l}\tau^{\mathcal{X}}_{-i,l}} \mathcal{E}^{-i,l}_{ik'} + \sum_{l\in\mathcal{K}\backslash\{k\}} \frac{\tau^{\mathcal{M}}_{il}-1}{\tau^{\mathcal{M}}_{il}} F_{il}\mathcal{E}^{il}_{ik'} \right\} d\log \tau^{\mathcal{M}}_{ik'} \tag{74}$$

We rewrite the welfare change as

$$\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1} dW_i = dR_i + \overbrace{\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1} \left(\sum_{j\neq i} \left[\frac{U_{ic,j}}{P^C_j}\right] dR_j + \sum_j \left[\frac{U_{ic,j}}{P^C_j}\right] w_j L_j d\log w_j \right)}^{\text{income term } \left[\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\right] d\mathcal{I}_i} \tag{75}$$

$$- \sum_j \left[\frac{U_{ic,i}}{P^C_i}\right]^{-1} \left[\frac{U_{ic,j}}{P^C_j}\right] C_j d\log P^C_j - \sum_j \left[\frac{U_{ic,i}}{P^C_i}\right]^{-1} \left[\frac{U_{im,j}}{P^M_j}\right] M_j d\log P^M_j$$

Collecting terms yields

$$\left[\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\right]^{-1}\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1} dW_i = d\mathcal{I}_i +$$

$$\sum_{k\in\mathcal{K}_i}\Bigg\{1+\frac{\tau^{\mathcal{X}}_{-i,k}-1}{\tau^{\mathcal{X}}_{-i,k}}(\mathcal{E}^{-i,k}_{-i,k}-1)+\left[\frac{F_{-i,k}}{\tau^{\mathcal{M}}_{-i,k}}\right]^{-1}\sum_{l\in\mathcal{K}_i\setminus\{k\}}\frac{F_{-i,l}}{\tau^{\mathcal{M}}_{-i,l}}\frac{\tau^{\mathcal{X}}_{-i,l}-1}{\tau^{\mathcal{X}}_{-i,l}}\mathcal{E}^{-i,l}_{-i,k} \tag{76}$$

$$-\tau^{\mathcal{M}}_{-i,k}\sum_j\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1}\left[\frac{U_{ic,j}}{P^C_j}\right]\left(S^E_{jk}S^C_{jk}+\sum_{k'\in\mathcal{K}_{-i}}S^{\Omega}_{k'k}\mathcal{C}^C_{jk'}\right)$$

$$-\tau^{\mathcal{M}}_{-i,k}\sum_j\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1}\left[\frac{U_{im,j}}{P^M_j}\right]\left(S^E_{jk}S^M_{jk}+\sum_{k'\in\mathcal{K}_{-i}}S^{\Omega}_{k'k}\mathcal{C}^M_{jk'}\right)\Bigg\}d\log\tau^{\mathcal{X}}_{-i,k}$$

$$-\tau^{\mathcal{M}}_{-i,k}\sum_j\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1}\left[\frac{U_{ic,j}}{P^C_j}\right]\sum_{j'}\sum_{k'}\mathcal{C}^C_{jk'}S^{\Omega}_{k'j'}\left[\frac{w_{j'}L_{j'}}{F_{-i,k}}\right]d\log w_{j'}$$

$$-\tau^{\mathcal{M}}_{-i,k}\sum_j\left[\frac{U_{ic,i}}{P^C_i}\right]^{-1}\left[\frac{U_{im,j}}{P^M_j}\right]\sum_{j'}\sum_{k'}\mathcal{C}^M_{jk'}S^{\Omega}_{k'j'}\left[\frac{w_{j'}L_{j'}}{F_{-i,k}}\right]d\log w_{j'}$$

Define factor centrality by

$$\mathcal{C}^A_{jj'}=\sum_k\mathcal{C}^A_{jk}\mathcal{S}^{\Omega}_{kj'} \tag{77}$$

to obtain the formula listed in the proposition.

D.2 Alliances

We consider the case of alliances in partial equilibrium with heterogeneous cost of military across countries. We abstract from general equilibrium considerations in this section; alliances, however, are derived in full generality.

We define effective military good by ζ that depends on a vector of militaries $\mathbf{m}$ via function $\mathcal{M}$ with an alliance jacobian $\mathcal{A}$.

$$\boldsymbol{\zeta}=\mathcal{M}(\mathbf{m}),\quad \frac{\partial\boldsymbol{\zeta}}{\partial\mathbf{m}}=\mathcal{A}$$

We define the aggregator $\bar{\zeta}$ that determines the final share of the military prize impicitly, so that

$$\gamma_i\left(\zeta_i/\bar{\zeta}\right)\beta-P^M_i m_i\to\max,\quad \sum_i\gamma_i(\zeta_i/\bar{\zeta})=1.$$

The resulting first-order condition is

$$\frac{1}{\bar{\zeta}}\sum_i\gamma'_i d\zeta_i=\left[\frac{1}{\bar{\zeta}^2}\sum_i\gamma'_i\zeta_i\right]d\bar{\zeta},\quad \frac{d\bar{\zeta}}{d\zeta_i}=\frac{\gamma'_i\bar{\zeta}}{\sum_k\gamma'_k\zeta_k},\quad \frac{d\ \log\bar{\zeta}}{d\log\zeta_i}=\frac{\gamma'_i\zeta_i}{\sum_k\gamma'_k\zeta_k}$$

If one plugs in $\bar{\zeta} = \mathcal{H} \sum_k \gamma'_k \zeta_k$ and takes a derivative with respect to ζ_j, one obtains

$$\sum_{i \neq j} \gamma'_i \frac{\zeta_i \mathcal{H} \gamma'_j}{\bar{\zeta}^2} - \gamma'_j \frac{\mathcal{H} \sum_k \gamma'_k \zeta_k - \zeta_j \mathcal{H} \gamma'_j}{\bar{\zeta}^2} = 0$$

That means that all indices of such type sum to a constant. The question is to find $\mathcal{H}$ that makes it sum to 1. We redefine $\tilde{\gamma} = \gamma \circ \mathcal{H}$ and $\tilde{\bar{\zeta}} = \bar{\zeta}/\mathcal{H}$. Then the identity would continue to hold. Hence, we can normalize $\mathcal{H} = 1$.

Define $g_i = \left[\sum_j a_{ij} \gamma'_j\right]^{-1}$, $\bar{g} = \sum_j g_j$, $\overline{P^M} = \sum_j (g_j/\bar{g}) P^M_j$. Then the FOCs are:

$$\gamma'_i \frac{a_{ii} - \zeta_i/(g_i \bar{\zeta})}{\bar{\zeta}} = \frac{P^M_i}{\beta}$$

$$\gamma'_i \zeta_i = \bar{\zeta} g_i \left[\gamma'_i a_{ii} - P^M_i \frac{\bar{\zeta}}{\beta}\right]$$

Summing over i yields

$$1 = \sum_i g_i \gamma'_i a_{ii} - g_i P^M_i \frac{\bar{\zeta}}{\beta}$$

$$\bar{\zeta} = \frac{\sum_j \gamma'_j a_{jj} g_j}{\sum_j g_j P^M_j} \beta, \quad \bar{\zeta} = \frac{\bar{G} - 1}{\bar{g} \overline{P^M}} \beta$$

That gives us the optimal military capacity

$$\zeta_i = g_i \left(a_{ii} - \frac{\bar{G} - 1}{\bar{g} \overline{P^M}} \frac{P^M_i}{\gamma'_i}\right) \frac{\bar{G} - 1}{\bar{g} \overline{P^M}} \beta, \quad \frac{\zeta_i}{\bar{\zeta}} = g_i \left(\gamma'_i a_{ii} - \frac{\bar{G} - 1}{\bar{g}} \frac{P^M_i}{\overline{P^M}}\right)$$

To back out optimal military good purchases, note that

$$\mathbf{m} = \mathcal{M}^{-1}(\boldsymbol{\zeta})$$

Some helpful derivatives for future work are:

$$\frac{\partial \overline{P^M}}{\partial P^M_j} = g_j/\bar{g}$$

$$\frac{\partial \bar{\zeta}}{\partial P^M_j} = -g_j \frac{\bar{G} - 1}{\bar{g}^2 \overline{P^M}^2} \beta = -\frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}}$$

$$\frac{\partial (\zeta_i/\bar{\zeta})}{\partial P^M_j} = g_i g_j \frac{\bar{G} - 1}{\bar{g}^2 \overline{P^M}^2} P^M_i = \frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}} \frac{g_i P^M_i}{\beta}$$

$$\frac{\partial (\zeta_j/\bar{\zeta})}{\partial P^M_j} = -g_j \frac{\bar{G} - 1}{\bar{g}} \frac{\overline{P^M} - (g_j/\bar{g}) P^M_j}{\overline{P^M}^2} = -\frac{g_j \bar{\zeta}}{\beta} + \frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}} \frac{g_j P^M_j}{\beta}$$

$$\frac{\partial \zeta_i}{\partial P^M_j} = \left[\frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}} \frac{g_i P^M_i}{\beta}\right] \bar{\zeta} + \zeta_i/\bar{\zeta} \left[-\frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}}\right] = \left[\frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}}\right] \left[\frac{g_i P^M_i \bar{\zeta}}{\beta} - \frac{\zeta_i}{\bar{\zeta}}\right]$$

$$\frac{\partial \zeta_j}{\partial P^M_j} = \left[-\frac{g_j \bar{\zeta}}{\beta} + \frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}} \frac{g_j P^M_j}{\beta}\right] \bar{\zeta} + \zeta_j/\bar{\zeta} \left[-\frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}}\right] = \left[\frac{g_j \bar{\zeta}}{\bar{g} \overline{P^M}}\right] \left[\frac{g_j P^M_j \bar{\zeta}}{\beta} - \frac{\zeta_j}{\bar{\zeta}}\right] - \frac{g_j \bar{\zeta}^2}{\beta}$$

D.3 Jacobian calculation

The goods market clearing can be written as

$$\mathbf{X} = \tilde{\mathbf{\Psi}}'(\mathbf{s^C}(\mathbf{wL} + \mathbf{R} + \mathbf{D} - \mathbf{M}) + \mathbf{s^M M}) \tag{78}$$

Note that

$$R_i = \sum_{k \in \mathcal{K}_{-i}} \frac{\tau_{ki}^{\mathcal{X}} - 1}{\tau_{ki}^{\mathcal{X}}} \frac{\Omega_{ki}^M}{\tau_{ki}^{\mathcal{M}}} X_k + \sum_{k \in \mathcal{K}} \frac{\tau_{k,-j}^{\mathcal{M}} - 1}{\tau_{k,-j}^{\mathcal{M}}} \Omega_{k,-j}^M X_k, \tag{79}$$

which can be recast in matrix form as $\mathbf{R} = \mathbf{\Lambda^R X}$. After accounting for revenue amplification, the goods market clearing condition can be recast as

$$\mathbf{X} = \mathbf{\Lambda^X} \tilde{\mathbf{\Psi}}'(\mathbf{s^C}(\mathbf{wL} + \mathbf{D} - \mathbf{M}) + \mathbf{s^M M}), \quad \mathbf{\Lambda^X} \equiv (\mathbf{I} - \tilde{\mathbf{\Psi}}'\mathbf{s^C \Lambda^R})^{-1}. \tag{80}$$

The factor market clearing is

$$\mathbf{wL} = \mathbf{\Omega^{L}}'\mathbf{X}. \tag{81}$$

Plugging in an expression for $\mathbf{X}$ yields

$$\mathbf{\Lambda^L wL} = \mathbf{\Omega^L}'\mathbf{\Lambda^X}\tilde{\mathbf{\Psi}}'(\mathbf{s^C}(\mathbf{D} - \mathbf{M}) + \mathbf{s^M M}), \quad \mathbf{\Lambda^L} \equiv \mathbf{I} - \mathbf{\Omega^L}'\mathbf{\Lambda^X}\tilde{\mathbf{\Psi}}'\mathbf{s^C}. \tag{82}$$

Solving for that equation allows us to solve for factor prices.

To find a wage jacobian, we now consider an equation that results from small policy changes:

$$\begin{aligned}(d\mathbf{\Lambda^L})\mathbf{wL} + \mathbf{\Lambda^L wL} d\log\mathbf{w} &= d\mathbf{\Omega^L}'\mathbf{\Lambda^X}\tilde{\mathbf{\Psi}}'(\mathbf{s^C}(-\mathbf{D} - \mathbf{M}) + \mathbf{s^M M}) \\ &+ \mathbf{\Omega^L}'(d\mathbf{\Lambda^X})\tilde{\mathbf{\Psi}}'(\mathbf{s^C}(-\mathbf{D} - \mathbf{M}) + \mathbf{s^M M}) \\ &+ \mathbf{\Omega^L}'\mathbf{\Lambda^X}(d\tilde{\mathbf{\Psi}}')(\mathbf{s^C}(-\mathbf{D} - \mathbf{M}) + \mathbf{s^M M}) \\ &+ \mathbf{\Omega^L}'\mathbf{\Lambda^X}\tilde{\mathbf{\Psi}}' d(\mathbf{s^C}(-\mathbf{D} - \mathbf{M}) + \mathbf{s^M M})\end{aligned} \tag{83}$$

One can further expand changes in each matrix:

$$d\tilde{\mathbf{\Psi}}' = -\tilde{\mathbf{\Psi}}'(d\tilde{\mathbf{\Omega}}')\tilde{\mathbf{\Psi}}', \qquad d\mathbf{\Lambda^X} = \mathbf{\Lambda^X} d(\tilde{\mathbf{\Psi}}'\mathbf{s^C \Lambda^R})\mathbf{\Lambda^X}. \tag{84}$$

Expressing the primitive $d\mathbf{\Omega}$ as a function of $d\log\mathbf{w}$ and $d\log\mathbf{P}$ for taxes and military changes allows to recast the expression as

$$\mathbf{A} d\log\mathbf{w} = d\log\mathbf{P}, \tag{85}$$

which allows us to recover a relevant jacobian.

D.4 Stockpiling

The utility contest function is

$$U_i(\{c_j\}_{i=1}^N, \{m_j\}_{i=1}^N) = c_i + \sum_{j\neq i} \alpha_{ij} c_j + \beta_i \frac{g(m_i)}{g(m_i) + \sum_{j\neq i} g(m_j)}. \tag{86}$$

Taking the first-order condition with two players yields

$$\frac{\beta_i}{P_i^M} \frac{g'(m_i)(g(m_i) + \sum_{j\neq i} g(m_j)) - g(m_i)g'(m_i)}{(g(m_i) + \sum_{j\neq i} g(m_j))^2} = \frac{1}{P_i^C} \tag{87}$$

or

$$\beta_i \frac{g'(m_i)}{g(m_i)} \frac{\nu_i(1-\nu_i)}{P_i^M} = \frac{1}{P_i^C}. \tag{88}$$

The derivative with respect to m_j is

$$\beta_i \frac{g'(m_j)}{g(m_j)} \frac{\nu_i \nu_j}{P_j^M}. \tag{89}$$

If $g(m_i) = m_i^\gamma$, then $g'/g = \gamma m_i^{-1}$. If there is a stockpile of goods m_{0i} and $g(m_i) = (m_{0i} + m_i)^\gamma$, then $g'/g = \gamma(m_{0i} + m_i)^{-1}$. We introduce $g'/g = \gamma \kappa_i m_i^{-1}$, where $\kappa_i = m_i/(m_{0i} + m_i)$ is the ratio of military goods to the total goods, including the stockpile. Note that in this case

$$\nu_i = \frac{(m_{0i} + m_i)^\gamma}{(m_{0i} + m_i)^\gamma + (m_{0,-i} + m_{-i})^\gamma} = \frac{\kappa_i^{-\gamma} m_i^\gamma}{\kappa_i^{-\gamma} m_i^\gamma + \kappa_{-i}^{-\gamma} m_{-i}^\gamma}. \tag{90}$$

For the purpose of γ estimation we assume that $\kappa_i = \kappa_{-i}$, simplifying the expression to

$$\nu_i = \frac{m_i^\gamma}{m_i^\gamma + m_{-i}^\gamma}. \tag{91}$$

From that,

$$\gamma \beta_i \kappa_i m_i^{-1} \frac{\nu_i(1-\nu_i)}{P_i^M} = \frac{1}{P_i^C}, \qquad \kappa_i \equiv \frac{m_i}{m_{0i} + m_i}, \tag{92}$$

naturally follows. The estimating equation can be rewritten as

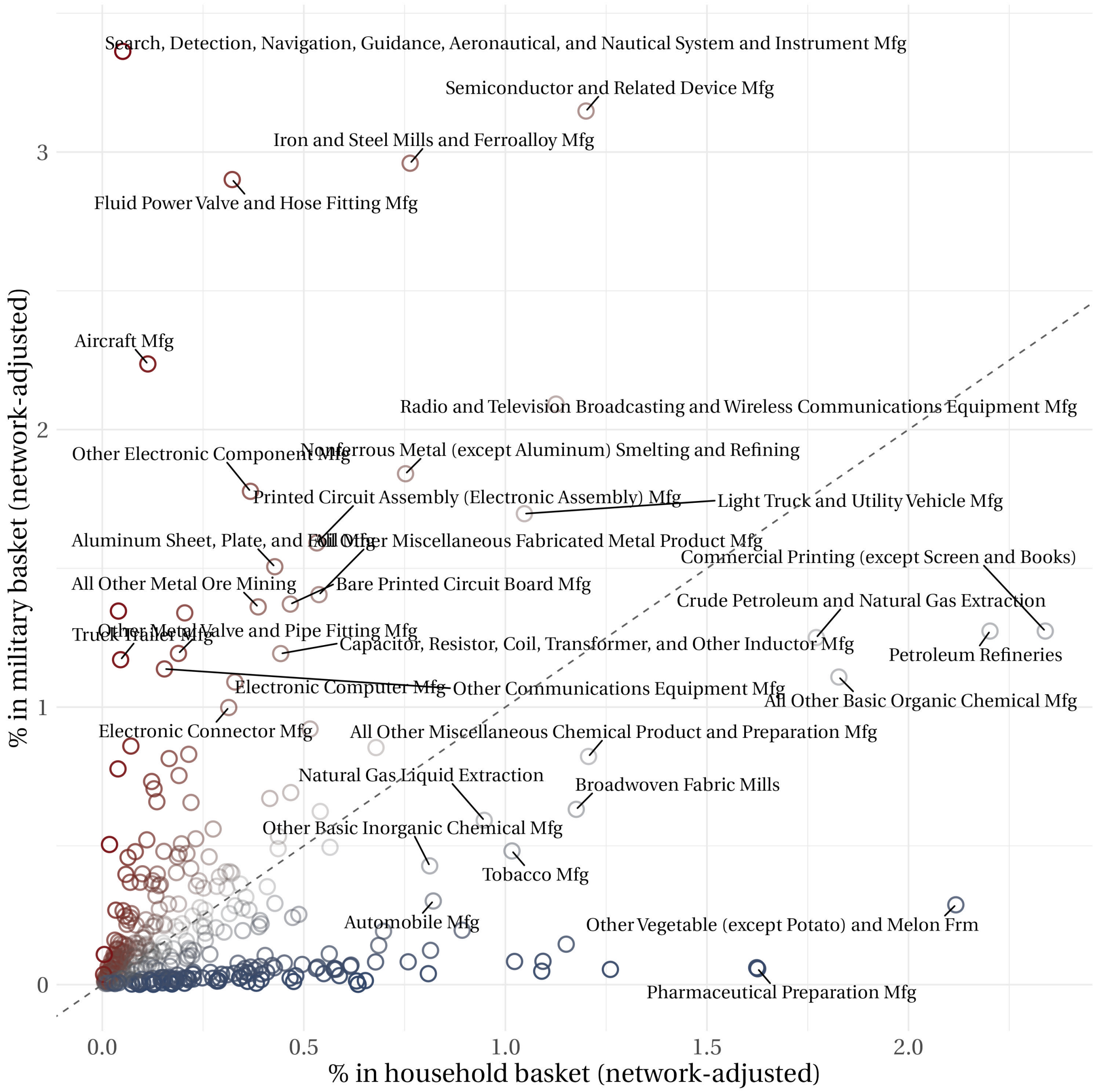
$$\log m_i = \log[\nu_i(1-\nu_i)] \tag{93}$$

D.5 Weights

General equilibrium conflict weight equals:

$$\beta_i = \frac{\frac{U_{i,ci}}{P_i^C} + \sum_j \frac{U_{i,cj}}{P_j^C} C_j \mathcal{J}_{M_i}^{P_j^C}}{\frac{U_{i,mi}/\beta_i}{P_i^M} - \sum_j \frac{U_{i,mj}/\beta_i}{P_j^M} M_j \mathcal{J}_{M_i}^{P_j^M}} = \frac{\frac{1}{P_i^C} + \sum_j \frac{\alpha_{ij}}{P_j^C} C_j \mathcal{J}_{M_i}^{P_j^C}}{\frac{g'(m_i)}{g(m_i)} \frac{\nu_i(1-\nu_i)}{P_i^M} + \sum_j \frac{g'(m_j)}{g(m_j)} \frac{\nu_i \nu_j}{P_j^M} M_j \mathcal{J}_{M_i}^{P_j^M}} \tag{94}$$

D.6 Production data for China



Notes: Data for the Chinese input-output table are taken for 2018 from the National Bureau of Economic Statistics. Data for the final military demand come from the revenue of military firms accessed via CSMAR. We convert the NBES industry classifications to NAICS (Rev. 2012). The consumer and military network-adjusted sales are calculated using Leontief inverses as $\left[\frac{(\mathbf{I}-\mathbf{\Omega})^{-1\prime}\mathbf{s}^{\mathbf{C}}}{[(\mathbf{I}-\mathbf{\Omega})^{-1\prime}\mathbf{s}^{\mathbf{C}}]^{\prime}\mathbf{1}}\right]$ and $\left[\frac{(\mathbf{I}-\mathbf{\Omega})^{-1\prime}\mathbf{s}^{\mathbf{M}}}{[(\mathbf{I}-\mathbf{\Omega})^{-1\prime}\mathbf{s}^{\mathbf{M}}]^{\prime}\mathbf{1}}\right]$, where $\mathbf{\Omega}$ is an input-output expenditure matrix and $\mathrm{s}^{\mathbf{C}}$, $\mathrm{s}^{\mathbf{M}}$ are expenditure shares of final agents.

Figure OA.D.1: 2018 input-output table for China

D.7 Weight β on military contest: A GE decomposition

Weight β_i for the U.S. increases from 180% to 250%, while the effect for China drops from 225% to 140%. This occurs because military spending affects demand for factors across countries, which affects final goods' prices. An increase in Chinese military demand lowers domestic wages ($d \log w_{\text{CHN}}/dM_{\text{CHN}} = -0.025$) because military sectors depend more on the Rest of the World than consumer sectors do (38% and 25.8% of the basket respectively; Table OA.D.1). The opposite occurs in the U.S. ($d \log w_{\text{CHN}}/dM_{\text{CHN}} = 0.017$, 31% and 19% of the basket). This results in the value of the prize being lower compared to the partial equilibrium in China and higher in the U.S..

	CHN			USA			ROW		
	w	P^C	P^M	w	P^C	P^M	w	P^C	P^M
CHN	-2.4847	-1.8049	-1.4985	0.0615	0.0740	0.0635	-0.8000	-0.5867	-0.4875
USA	-0.2051	-0.2245	-0.2136	1.6931	1.1047	1.3337	-0.3901	-0.2833	-0.3238
ROW	0.0000	-0.0951	0.0000	0.0000	0.0473	0.0000	0.0000	-0.0392	0.0000

Notes: The rows indicate the countries that increase their military spending. The columns report price reactions in the respective countries. The rest-of-the-world wage is normalized.

	CHN		USA		ROW	
	C	M	C	M	C	M
CHN	72.498	60.181	3.659	2.101	3.608	0.000
USA	1.739	1.565	65.115	78.693	2.662	0.000
ROW	25.763	38.254	31.226	19.206	93.730	100.000

Notes: The rows report the network-adjusted purchase share of the labor factor across various countries by consumers and military.

Table OA.D.1: Decomposition of general equilibrium effects behind military spending

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